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Predicting Solar Cycle 24 With a Solar Dynamo Model

Arnab Rai Choudhuri,[1,2,*] Piyali Chatterjee,[1] and Jie Jiang[2]

[1]*Department of Physics, Indian Institute of Science, Bangalore-560012, India*
[2]*National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China*

Whether or not the upcoming cycle 24 of solar activity will be strong is being hotly debated. The solar cycle is produced by a complex dynamo mechanism. We model the last few solar cycles by "feeding" observational data of the Sun's polar magnetic field into our solar dynamo model. Our results fit the observed sunspot numbers of cycles 21–23 reasonably well and predict that cycle 24 will be about 35% weaker than cycle 23.

Solar activity affects our space environment, thereby influencing various aspects of human life [1]. So it is vitally important to develop capabilities for predicting strengths of the 11-year cycles of solar activity. It has been believed for some time that the Sun's polar magnetic field at the preceding minimum gives an indication of the strength of the next solar cycle [2]. The weakness of the present polar field has already led to predictions that cycle 24 will be the weakest cycle in 100 years [3,4]. Since the solar cycle is produced by a dynamo mechanism, one would like to make a prediction of cycle 24 from a detailed solar dynamo model also. The only previous dynamo-based prediction is that cycle 24 will be one of the strongest cycles [5]. Our aim is to generate an independent prediction for cycle 24 from a different dynamo model by a different methodology. We identify the Babcock-Leighton process for poloidal field generation as the main source of randomness in solar cycles. A theoretical mean field model of the solar dynamo produces a poloidal field at the end of a cycle which would be typical of an "average" solar cycle. In order to model actual solar cycles, a theoretical mean field model of the solar dynamo has to be "corrected" by feeding actual observational data of poloidal field. Since such data are available only from the mid-1970s, this method can be used to model solar cycles only from that time. We carry on our calculations by feeding the DM (Dipole Moment) values of solar polar field computed by Svalgaard *et al.* [3] into our already published solar dynamo model [6,7].

Current solar dynamo models combine three basic processes. (i) The strong toroidal field is produced by the stretching of the poloidal field by differential rotation in the tachocline. (ii) The toroidal field generated in the tachocline gives rise to active regions due to magnetic buoyancy, and the decay of tilted bipolar active regions produces the poloidal field by the Babcock-Leighton mechanism. (iii) The meridional circulation advects the poloidal field first to high latitudes and then down to the tachocline. Two-dimensional mean field dynamo models based on these three processes were first constructed about a decade ago [8,9]. We believe that the processes (i) and

(iii) are reasonably ordered and deterministic. In contrast, the process (ii) involves an element of randomness, which presumably is the primary cause of solar cycle fluctuations. First, although active regions appear in a latitude belt at a certain phase of the solar cycle, where exactly within this belt the active regions appear seems random. Second, there is considerable scatter in the tilts of bipolar active regions around the average given by Joy's law. The action of the Coriolis force on the rising flux tubes gives rise to Joy's law [10], whereas convective buffeting of the flux tubes in the upper layers of the convection zone causes the scatter of the tilt angles [11]. Since the poloidal field generated from an active region by the Babcock-Leighton process depends on the tilt, the scatter in the tilts introduces a randomness in the poloidal field generation process.

The poloidal field gets built up during the declining phase of the cycle and at the minimum, when there are no sunspots, we have the polar field cumulatively produced from the sunspots during the previous cycle. The polar field at the solar minimum produced in a mean field dynamo model is some kind of "average" polar field during a typical solar minimum. The polar field during a particular solar minimum may be stronger or weaker than this average field. We propose the following methodology for modelling the solar cycles with a mean field dynamo model. We run the dynamo code in the usual way from one solar minimum to the next. Then, at the time of the minimum, we change the amplitude of the polar field suitably to make it agree with the observed value of the polar field and run the code again to the next minimum. Proceeding in this way, we can correct for the randomness introduced in the Babcock-Leighton mechanism by using actual observational data.

Our calculations are based on the solar dynamo code SURYA. This code, along with a detailed guide [12], is freely available for use by solar physicists. Full details of the two-dimensional kinematic dynamo model which is solved by SURYA are available elsewhere [7,12]. In what was referred to as the *standard model* in Sect. 4 of Chatterjee *et al.* [7], we change some parameters to make the period of the solar cycle equal to 10.6 years (the period

in the standard model was 14 years). The old values and
changed values of the parameters are listed in Table I.

We now discuss how we change the value of the polar
field during successive solar minima to feed the relevant
information about the past cycles into the code. Reliable
data about polar fields from Wilcox Solar Observatory
(WSO) and Mount Wilson Observatory (MWO) exist
only for the minima at the ends of the cycles 21, 22, and
23. Additionally, MWO data exist for one previous mini-
mum (at the end of cycle 20), though the quality of data
was not so good at that time. Svalgaard *et al.* [3] have
analyzed these data carefully and come up with a parame-
ter for the solar minima, which they call "Dipole Moment
(DM)." This DM, which is actually a proxy of the Sun's
dipole moment and is a good measure of the polar field
during the solar minimum, has its values for the last 3 solar
minima listed in Table I of Svalgaard *et al.*, Fig. 3. From
Fig. 3 of their paper, we estimate DM for the previous
minimum at the end of cycle 20 to be about 250 μT,
although the data appear noisy. According to Table I,
values of DM at the ends of cycles 21, 22, and 23 are
respectively 245.1 μT, 200.8 μT, and 119.3 μT. The next
question we have to address is: what value of DM corre-
sponds to the polar field of an "average" cycle? This
question is not so straightforward to settle, given the fact
that there is a trend of cycle amplitudes steadily increasing
since the Maunder minimum [13]. We tentatively take
cycle 23 as an average cycle and the value of DM before
its beginning (which is 200.8 μT) denoted by $\overline{\text{DM}}$ as the
average for a typical average solar minimum. If we divide
the DM value of a particular minimum by $\overline{\text{DM}} = 200\ \mu$T,
we get a numerical factor which we would call γ. The
values of γ at the ends of cycles 20, 21, 22, and 23 are,
respectively, 1.25, 1.23, 1.0, and 0.60.

The poloidal field in a two-dimensional dynamo prob-
lem is described by a scalar function $A(r, \theta)$. From a
regular run of the dynamo code, we can find out the value
of the amplitude of A at the solar minimum, which would
correspond to an "average" value for a typical solar mini-
mum. Let us call this $\bar{A}_{\min}$. Suppose we run the dynamo
code till a solar minimum for which we know the value of
γ from observational data. At all grid points above $0.8R_\odot$,

we multiply A by a constant factor such that the amplitude
of A becomes $\gamma \bar{A}_{\min}$. We do not make any changes in the
values of A below $0.8R_\odot$. This ensures that the poloidal
field in the upper layers, which has been created by the
Babcock-Leighton mechanism operating during the last
cycle, gets corrected to the observed value, whereas the
poloidal field at the bottom of the convection zone, which
may have been created during the still earlier cycles, is left
unchanged. After changing A above $0.8R_\odot$ in this fashion,
we run the code till the next minimum when this procedure
is repeated.

Since we have values of γ at the ends of cycles 20–23,
our procedure for generating a forecast for cycle 24 is now
straightforward. We take a relaxed solution of our dynamo
code which has been stopped at a solar minimum. We
identify this minimum as the minimum at the end of cycle
20 and change the values of A above $0.8R_\odot$ in accordance
with the value of γ (which is 1.25). Then we run the code
till the next minimum and again change the values of A
above $0.8R_\odot$. Doing this thrice, we come to the minimum
at the end of the cycle 23. The next run after this generates
the forecast for cycle 24. It may be noted that the poloidal
field lines become somewhat discontinuous at $r = 0.8R_\odot$
after we change the values of A above $0.8R_\odot$ in accordance
with observational data. This discontinuity can be seen in
Fig. 1 where we plot the poloidal field lines at the minimum
before cycle 24 just after updating the values of A.
However, we find that this discontinuity gets smoothed
out within a time scale of weeks. In the particular dynamo
model we have used, we find that our results are not
significantly different even if we update the values of A
at the minima all the way to the bottom of the convection

TABLE I. The original values of the parameters in the *stan-
dard model* (Sect. 4 of Chatterjee *et al.* [7]) along with the
changed values we use now. The first four parameters control the
amplitude, penetration depth, equatorial return flow thickness
and the position of the inversion layer of the meridional circu-
lation, respectively. The tachocline width is denoted by d_{tac}.

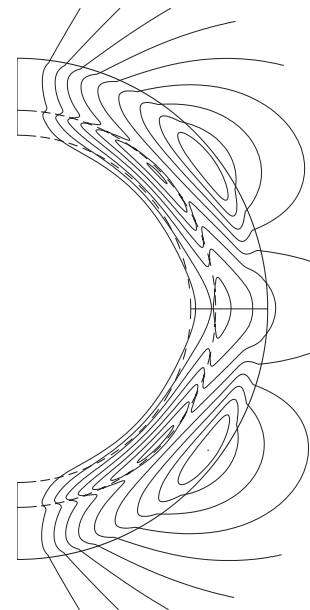| parameter	Standard Model	This Model
v_0	$-29\ \text{m s}^{-1}$	$-34\ \text{m s}^{-1}$
R_p	$0.61R_\odot$	$0.635R_\odot$
β_2	$1.8 \times 10^{-8}\ \text{m}^{-1}$	$1.3 \times 10^{-8}\ \text{m}^{-1}$
r_0	$0.1125R_\odot$	$0.1286R_\odot$
d_{tac}	$0.05R_\odot$	$0.03R_\odot$

FIG. 1.   A snapshot of streamlines of the poloidal field given by
constant contours of $Ar \sin\theta$ just after correcting by the DM
value for the poloidal field at the minimum before cycle 24. The
dashed lines correspond to $r = 0.7R_\odot$ and $r = 0.8R_\odot$.

zone, since we have a higher diffusivity compared to some
of the other models (see [5]), and not much poloidal field
generated in the previous cycles is left at the bottom of the
convections zone. The situation may be different for other
combination of parameters and one should leave the values
of A at the bottom of the convection zone unchanged
during the updating, to make sure that the memories of
the previous cycles are in principle not erased.

Before presenting results obtained with actual observed
values of DM fed into the code, we present some results
obtained by changing the poloidal field arbitrarily during a
solar minimum and then running the code without any
further interruptions. Figure 2 gives sunspot number plots
obtained by increasing and decreasing the poloidal field by
30% above $0.8R_\odot$ at a solar minimum. We find that the next
two solar minima are both affected, after which the mem-
ory of the poloidal field change gets lost. Conversely, we
expect the strength of a maximum to depend on polar fields
in the two preceding minima.

Figure 3 now presents our results for cycles 21–24
generated by our methodology. The top panel superposes
the sunspot number generated from our model on the
observational data. The middle panel gives the B_r at a
latitude of 70° obtained from the dynamo model, showing
the jumps at the solar minima when we change the poloidal
field in accordance with the observed value of DM. The
bottom panel shows the butterfly diagram produced by our
model. We see in the top panel that the theoretical plot is in
reasonably good agreement with the observational data for
cycles 21–23, whereas cycle 24 comes out as the weakest
cycle in a long time. Since the value of DM during the
minima at the ends of cycles 22 and 23 is lower than the
values of DM in the two preceding minima, the weakness
of cycle 24 appears like a very robust result, which does not
change with small changes in the parameters of the prob-
lem such as the chosen value of $\overline{DM}$. We may point out that
we calculate the number of sunspot eruptions theoretically
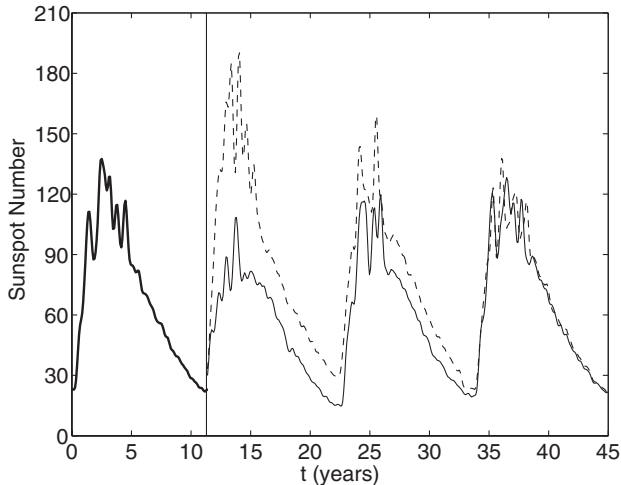by assuming the toroidal field B to be buoyant whenever it

FIG. 2.   Monthly smoothed sunspot number plots by increasing
(dashed line) and decreasing (solid line) the poloidal field by
30% above $0.8R_\odot$ at a solar minimum (indicated by the vertical
line), based on our model. The polar field takes some time to be
advected to the midlatitudes in the tachocline where a strong
toroidal field is produced during the solar maxima. The advec-
tion time along the particular streamline which passes through
the middle of the tachocline should be the appropriate advection
time to be considered. Since the meridional circulation is slightly
more penetrating in our model (see [6,7]) compared to Dikpati &
Gilman [5], the velocities along this particular steamline are
larger in our model, although the overall velocity amplitudes at
the base of the convection zone are comparable in both the
models. Thus the advection time in the Dikpati-Gilman model is
somewhat longer than that in our model, which is of order 10 yr.
After a minimum at the end of cycle n, the maxima of the next
two cycles $n+1$ and $n+2$ come about 5 yr and 16 yr later,
respectively. Since the advection time in our model is almost an
arithmetic mean between these two, the two next maxima are
both affected in our model. In the Dikpati-Gilman model with
the longer advection time, the poloidal field of cycle n seems
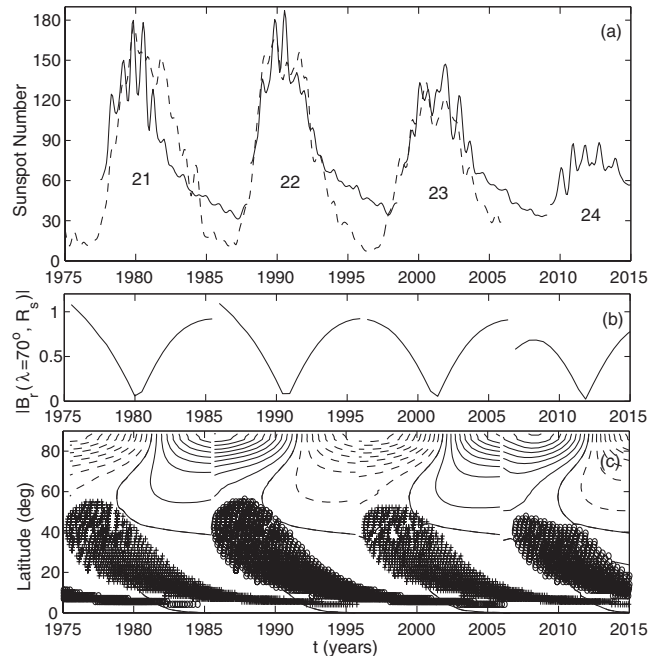primarily to affect the cycle $n+2$ rather than the cycle $n+1$.

FIG. 3.   Results for cycles 21–24. (a) The theoretical monthly
smoothed sunspot number (solid line) superposed on the
monthly smoothed sunspot numbers from observation (dashed
line). (b) A plot of B_r at the surface at a latitude of 70°. (c) The
theoretical butterfly diagram, with contours of B_r at the surface
in the time-latitude plot. In the middle panel, it is interesting to
note that, even though we change B_r abruptly at a minimum, by
the next minimum, its value relaxes to values close to what
would be the ''average'' value for a typical cycle. The cycles 21
and 22, which are of comparable strength, are easy to model as
we see in the top panel, since they follow solar minima having
DM of comparable values. The cycle 23, which follows a mini-
mum of low DM, is weak. The cycle 24 is even much weaker.
The theoretical plot in the top panel was generated by using
$\overline{DM} = 200\ \mu$T. The theoretical plots are found to be qualita-
tively similar when we take $\overline{DM}$ in the range 150–220 μT.

is higher than a critical value B_c within the convection zone (discussed in detail in [7,12]). This procedure seems to introduce an automatic noise in the theoretical sunspot number, as seen in Figs. 2 and 3(a). Since a finer grid leads to eruptions from a larger number of points, the absolute value of the theoretical sunspot number from our numerical code does not have any particular significance. To generate Fig. 3(a), we scaled the theoretical sunspot number suitably to make it fit the observational plot.

A future paper will present calculations in which instead of DM we feed magnetogram data at different latitudes during solar minima into our model. While the use of more detailed polar field data may lead to more realistic predictions, the attractiveness of the scheme presented in this Letter is that it is extremely straightforward to implement and is probably reasonably reliable, as we have been able to model cycles 21–23 reasonably well during the rising and the maximum phases. The fit during the declining and the minimum phases is not so good, since the best theoretical model we have been able to construct keeps producing a few sunspots at low latitudes even during the minima [See Fig. 3(c)]. However, the sunspots at low latitudes have small tilts and do not contribute much to the Babcock-Leighton process. Hence this unsatisfactory aspect of our theoretical model will not introduce much error in the poloidal field generation calculation.

Since the dominant processes during the rising phase of a cycle from a minimum to a maximum are fairly regular processes like the magnetic field advection and toroidal field generation by differential rotation, a good knowledge of magnetic configurations during a minimum should enable a good theoretical model to predict the next maximum reliably. On the other hand, the dominant process in the declining phase of a cycle is the poloidal field generation by the Babcock-Leighton process which involves randomness and cannot be predicted in advance by theoretical models. In other words, we suggest that the rising phase of the cycle is predictable (enabling us to predict the strength of the maximum a few years ahead of time), but the declining phase is *not* predictable. So, it may never be possible to make a realistic prediction of a maximum more than 7–8 years ahead of time, even when we have better theoretical models and better magnetic data.

Although our forecast is in agreement with physical intuition as well as forecasts based on polar field strength [3,4], it is completely opposite of the only other forecast based on a detailed dynamo model [5]. The methodology used by Dikpati & Gilman [5] for feeding the observational data in the theoretical model differs from ours at a fundamental conceptual level. They use the sunspot area as the source term for the generation of the poloidal field, whereas the tacit assumption behind our methodology is that the poloidal field generation involves randomness and cannot be calculated deterministically from the past sunspot data. Cycles with many sunspots do not necessarily

produce strong poloidal fields at the end. This is clearly seen in the analysis of Makarov *et al.* [14] (see their Fig. 1) who have used the positions of Hα filaments to estimate polar fields for the better part of a century. They find that the polar field during a minimum is correlated with the strength of the next cycle, but the strength of the cycle has no good correlation with the polar field produced at its end. A similar conclusion also follows from the polar faculae data [15]. If our identification of the poloidal field generation by the Babcock-Leighton process as the main source of randomness in the solar dynamo is correct, then the methodology of Dikpati & Gilman [5] should in principle not work, although they claim to ''predict'' many past cycles correctly. Since their forecast for cycle 24 is completely opposite of ours, it should become apparent in the next 4–5 years as to which forecast comes closer to truth.

*Electronic address: arnab@physics.iisc.ernet.in
[1] S. Clark, Nature (London) **441**, 402 (2006).
[2] K. H. Schatten, P. H. Scherrer, L. Svalgaard, and J. M. Wilcox, Geophys. Res. Lett. **5**, 411 (1978).
[3] L. Svalgaard, E. W. Cliver, and Y. Kamide, Geophys. Res. Lett. **32**, L01104 (2005).
[4] K. H. Schatten, Geophys. Res. Lett. **32**, L21106 (2005).
[5] M. Dikpati and P. A. Gilman, Astrophys. J. **649**, 498 (2006).
[6] D. Nandy and A. R. Choudhuri, Science **296**, 1671 (2002).
[7] P. Chatterjee, D. Nandy, and A. R. Choudhuri, Astron. Astrophys. **427**, 1019 (2004).
[8] A. R. Choudhuri, M. Schüssler, and M. Dikpati, Astron. Astrophys. **303**, L29 (1995).
[9] B. R. Durney, Sol. Phys. **160**, 213 (1995).
[10] S. D'Silva and A. R. Choudhuri, Astron. Astrophys. **272**, 621 (1993).
[11] D. Longcope and A. R. Choudhuri, Sol. Phys. **205**, 63 (2002).
[12] A. R. Choudhuri, *The User's Guide to the Solar Dynamo Code SURYA* (available upon request) (2005).
[13] D. H. Hathaway and R. M. Wilson, Sol. Phys. **224**, 5 (2004).
[14] V. I. Makarov, A. G. Tlatov, D. K. Callebaut, V. N. Obridko, and B. D. Shelting, Sol. Phys. **198**, 409 (2001).
[15] N. R. Sheeley, Astrophys. J. **374**, 386 (1991).