Origin of Grand Minima in Sunspot Cycles

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One of the most striking aspects of the 11-year sunspot cycle is that there have been times in the past when some cycles went missing, a most well-known example of this being the Maunder minimum during 1645–1715. Analyses of cosmogenic isotopes (^{14}C and ^{10}Be) indicated that there were about 27 grand minima in the last 11 000 yrs, implying that about 2.7% of the solar cycles had conditions appropriate for forcing the Sun into grand minima. We address the question of how grand minima are produced and specifically calculate the frequency of occurrence of grand minima from a theoretical dynamo model. We assume that fluctuations in the poloidal field generation mechanism and in the meridional circulation produce irregularities of sunspot cycles. Taking these fluctuations to be Gaussian and estimating the values of important parameters from the data of the last 28 solar cycles, we show from our flux transport dynamo model that about 1–4% of the sunspot cycles may have conditions suitable for inducing grand minima.

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A few years after the initiation of telescopic observations of sunspots in 1610, there was a period from 1645 to 1715 when very few sunspots appeared on the face of the Sun. This period is known as the Maunder minimum [1]. Although reliable sunspot data did not exist before 1610, the solar activity at earlier times can be studied from the analyses of the abundances of cosmogenic isotopes like ^{14}C in old tree rings [2,3] and ^{10}Be in polar ice [4]. When sunspots are absent, the magnetic field in the solar wind becomes weak and more galactic cosmic rays can reach Earth, producing more of such radioactive isotopes in the atmosphere. Analyses of these isotopes indicate that there have been about 27 grand minima of different durations in the last 11 000 years [5]. Since there were about 1000 solar cycles during this period, the occurrence of 27 grand minima implies that about 2.7% cycles had conditions appropriate for forcing the Sun into grand minima. Furthermore, this study showed that the Sun was in the grand minima state for about 17% of the time. We also mention that there is evidence that some solarlike stars show grand minima [6]. Therefore, it is very important to understand the physics of the origin of the grand minima and the probability of occurrence of such grand minima. Moreover, sunspot cycles have an important effect on the space environment and the Earth climate system [7,8]. Therefore, understanding the grand minima is also important to the space weather and Earth climate communities.

It appears that the most promising theoretical model for the solar cycle at the present time is the flux transport dynamo model [9–11], which has been reviewed recently by Charbonneau [12] and Choudhuri [13]. Let us consider the question of how irregularities arise in solar cycles. One important assumption of the flux transport dynamo model is that the poloidal field is produced by the Babcock-Leighton mechanism, in which tilted bipolar sunspots give rise to the poloidal field after their decay. The amount of poloidal field generated depends on the tilt angle of the bipolar sunspots. This tilt is produced by the action of the Coriolis force acting on the magnetic flux tube rising through the solar convection zone due to magnetic buoyancy [14], and the average tilt at a solar latitude is given by Joy's law. However, there is a large scatter in the tilt angles around the average given by Joy's law [15], presumably due to the effects of turbulence on the rising flux tubes [16]. Hence, the Babcock-Leighton mechanism has an inherent randomness, because of which the poloidal field generated at the end of a cycle would vary from one cycle to the other [17]. The second source of irregularities in solar cycles comes from the fluctuations in the meridional circulation of the Sun, which plays a crucial role in the flux transport dynamo. The period of the cycle in the theoretical model is approximately inversely proportional to the amplitude of the meridional circulation [10,18]. Presumably, the variations in the periods of past cycles were produced primarily by variations in the meridional circulation. Although we have direct measurements of the meridional circulation flow speed only during the last few years [19,20], the periods of past cycles can be used to draw inferences about meridional circulation variations in the past [21–24]. Such variations in meridional circulation also do cause variations in the strengths of different cycles in addition to variations in their periods. Suppose the meridional circulation has become weaker than usual. Then, the period of the cycle will be longer and diffusion will have more time to act on the magnetic field, making the cycle weaker. If diffusion is assumed in the range 10^{12}–10^{13} cm^2 s^{-1}, consistent with mixing length arguments [25] (which we do in our model), then this effect overcomes the opposing effect of differential rotation also getting more time to generate more toroidal field, and the cycles become weaker when the meridional circulation slows down [18,21,24]. However, in the model developed by the High Altitude Observatory group, the diffusion is taken to be about 50 times smaller than what we take [10,26], leading to the opposite effect of cycles getting stronger with slower meridional circulation due to the generation of more toroidal field by differential rotation over a longer time. Several arguments in favor of the higher diffusivity used by us are summarized in Sect. 5 of Jiang *et al.* [27].

In order to model grand minima theoretically, we have to run our theoretical flux transport dynamo model with fluctuations in poloidal field generation and fluctuations in meridional circulation. Some studies have shown that large fluctuations in the poloidal field generation mechanism or large fluctuations in the meridional circulation can force the dynamo into intermittencies resembling grand minima [28–33]. Karak [21] found that the dynamo is driven into a grand minimum if the poloidal field at the end of a cycle and the meridional circulation at that time fall to sufficiently low values. Defining a grand minimum as an absence of sunspots for at least 20 years (the same definition as used by Usoskin *et al.* [5] in estimating the number of grand minima in the past 11 000 years), the shaded region of Fig. 1 indicates the combined values of poloidal field and meridional circulation at the end of a cycle necessary for forcing the dynamo into grand minima according to our theoretical model. The important question now is to estimate the probability of this happening at the end of a cycle. We estimate this probability in the following way.

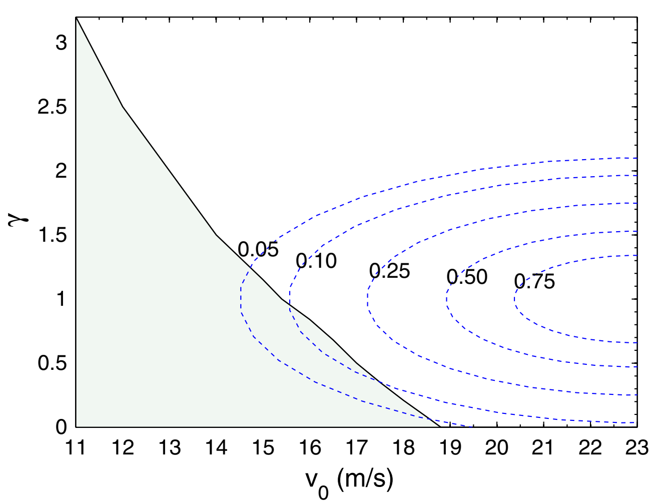

FIG. 1 (color online). The solid line shows the values of the meridional circulation amplitude v_0 and the poloidal field scale factor γ, which produce grand minima of duration ~20 yrs. The parameters lying in the shaded region produce grand minima of longer duration. The dashed curves are the contours of the joint probability $P(\gamma, v_0)$, with the values of $P(\gamma, v_0)$ (excluding the constant prefactor) given in the plot. The scale factor γ is defined as the amplitude of the poloidal field at the end of a cycle divided by its value in the absence of fluctuations. This figure is produced by using our theoretical solar dynamo model (details are given by Karak [21]).

Assuming that the inverse of the cycle period gives an approximate value of the meridional circulation amplitude during that cycle (as suggested by theoretical flux transport dynamo models [10,18]), Karak and Choudhuri [24] concluded that the meridional circulation has changed randomly in the past with a correlation time around 30–40 yrs (also see the similar study based on the low order dynamo model [22,23]). Figure 2(a) shows a histogram of the estimated values of the meridional circulation amplitude during the last 28 cycles. The solid curve in Fig. 2(a) shows the Gaussian having the mean $\overline{v_0} = 23\ \mathrm{m\,s^{-1}}$ and the standard deviation $\sigma_v = 3.34\ \mathrm{m\,s^{-1}}$ calculated from the data presented in the histogram. We find that the Gaussian is a reasonable fit, although we do not have any data points lying way out in the Gaussian tail. Jiang *et al.* [27] (see also [15]) pointed out on the basis of observational data that there is a good correlation between the poloidal field at the end of a cycle and the strength of the next cycle. Assuming a perfect correlation, we can use the strengths of the following cycles to obtain values γ of poloidal field at the ends of previous cycles (scaled by taking their average value as the unit). Figure 2(b) is a histogram of the values γ of poloidal field obtained in this way. It should, however, be kept in mind that variations in the meridional circulation also contribute to fluctuations of cycle strengths [18,21,24,34], and fluctuations of poloidal field at the end of cycles obtained by our method [as shown in Fig. 2(b)] are probably an overestimate. The solid curve in Fig. 2(b) is the Gaussian having the standard deviation $\sigma_\gamma = 0.35$ obtained from the data in the histogram (the mean is 1 by definition). We expect on general grounds the distribution of γ to follow a Gaussian, although we find that the Gaussian fit in Fig. 2(b) is not as good a fit as in

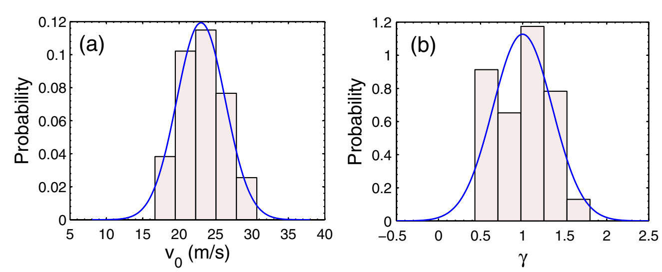

FIG. 2 (color online). (a) Histogram of the meridional circulation amplitude derived from the observed periods of the last 28 solar cycles. The solid curve is the Gaussian function with mean = 23 $\mathrm{m\,s^{-1}}$ and standard deviation $\sigma_v = 3.34\ \mathrm{m\,s^{-1}}$. (b) Histogram of the poloidal field scale factor derived from the peak sunspot number of the last 28 solar cycles. The solid curve is the Gaussian function with mean = 1 and standard deviation $\sigma_\gamma = 0.35$. The probabilities plotted along the vertical axis in both (a) and (b) are obtained by dividing the number of data points in a data bin by the total number of data points and the horizontal width of the data bin.

TABLE I. This table gives the values of the probability (in %) of the initiation of grand minima [given by $\int P(\gamma, v_0) d\gamma dv_0$ over the shaded region of Fig. 1] for different combinations of the standard deviations σ_v and σ_γ.

		σ_γ 0.25	0.35	0.46
σ_v	3.20	0.8	1.3	1.7
	3.34	0.9	1.3	1.9
	3.50	1.1	1.7	2.2

Fig. 2(a). This is not surprising, given that our data set comprised of only 28 cycles is quite small. If we assume that the fluctuations in the amplitude v_0 of meridional circulation at the solar surface and the fluctuations in γ both follow Gaussian distributions, then we can draw one obvious inference. The joint probability that the poloidal field at the end of a cycle lies in the range γ, $\gamma + d\gamma$ and that the amplitude of meridional circulation at the same time lies in the range v_0, $v_0 + dv_0$ is given by

$$P(\gamma, v_0) d\gamma dv_0 = \frac{1}{\sigma_v \sqrt{2\pi}} \exp\left[-\frac{(v_0 - \overline{v_0})^2}{2\sigma_v^2}\right] \times \frac{1}{\sigma_\gamma \sqrt{2\pi}} \exp\left[-\frac{(\gamma - 1)^2}{2\sigma_\gamma^2}\right] d\gamma dv_0.$$

In Fig. 1, we show various contours corresponding to different values of $P(\gamma, v_0)$ (excluding the constant prefactor). The probability that γ and v_0 at the end of a cycle jointly lie within a certain area in Fig. 1 is easily obtained by integrating $\int P(\gamma, v_0) d\gamma dv_0$ over that area. On carrying out this integration over the shaded region in Fig. 1 that corresponds to conditions for producing grand minima, we find the value of the integral to be 0.013 (i.e., 1.3%), remarkably close to the probability of occurrence of grand minima on the basis of observational data [5]. Table I gives the value of this integral for a few combinations of σ_v and σ_γ. It may be noted that there would be contributions to the probability from the regions to the left and to the bottom of the shaded region in Fig. 1. We have checked that these contributions are negligible.

To check whether grand minima really do occur in accordance with the above simple probability estimate, we have carried out extensive simulations on the basis of our dynamo code [11,35]. Karak [21] changed the values of a few parameters, and our present simulations are based on this model. We introduce fluctuations in poloidal field generation by the method proposed by Choudhuri *et al.* [17]. At the end of every cycle, we multiply the poloidal field above $0.8R_\odot$ by the random number γ, obeying the Gaussian distribution shown in Fig. 2(b). This procedure introduces fluctuations in the poloidal field generated in the last cycle lying in the upper portions of the convection zone, whereas any poloidal field produced at the earlier cycles lying at the bottom of the convection zone remains unchanged. While this procedure introduces a momentary discontinuity in the field lines at depth $0.8R_\odot$ (see Fig. 1 in Jiang *et al.* [17]), this discontinuity disappears soon and does not cause any problem. To introduce fluctuations in meridional circulation, we change its amplitude randomly after every 30 yrs such that the amplitudes obey the Gaussian distribution shown in Fig. 2(a). On running the code for 11 000 yrs in this way, we obtain the number of grand minima in the range 24–30 for different realizations of randomly generated meridional circulation and γ—remarkably close to the observational finding of 27 grand minima in the last 11 000 yrs. Another important result is that we find the Sun to spend about 10–15% of the time in a grand minimum state, which is very close to the 17% found in the observational study [5]. Figure 3 shows the durations of grand minima and their times of occurrence for a particular run. We ourselves have been amazed that the observational data are reproduced so well on running our code with the simple incorporation of the fluctuations suggested by the histograms in Fig. 2, without having to change any parameters of the dynamo model compared to our previous work. Figure 4 is a sample plot showing how

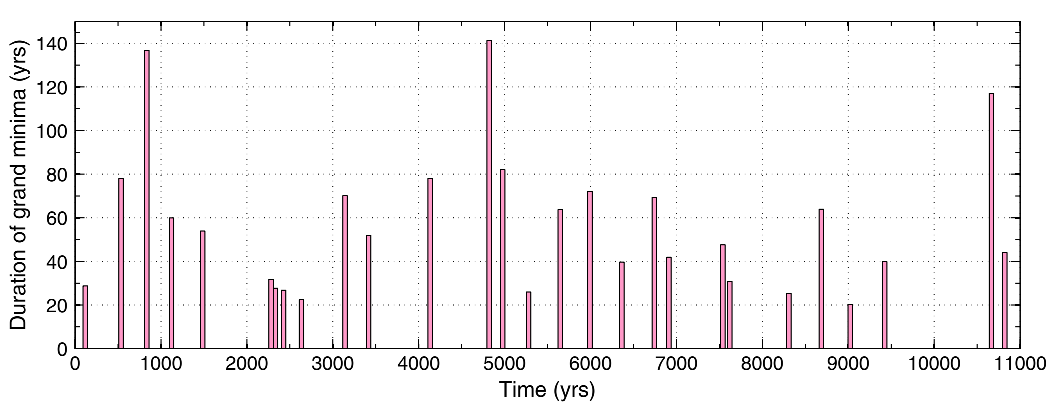

FIG. 3 (color online). The durations of grand minima indicated by vertical bars at their times of occurrence in a 11 000 yr simulation. This is the result of a particular realization of random fluctuations that produced 28 grand minima.

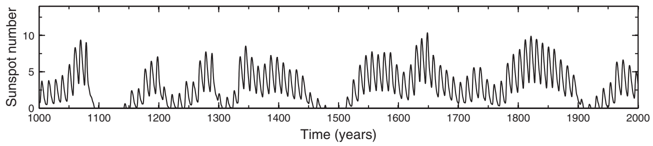

FIG. 4. The plot of sunspot number against time for a typical 1000 years. The two grand minima around 1100 and 1500 years can be seen in Fig. 3 as well.

the sunspot number varied for a typical 1000 years, during which two grand minima occurred. Histograms of the duration of grand minima and waiting time between grand minima are shown in Fig. 5. These histograms are to be compared with Figs. 7 and 6 in Usoskin *et al.* [5]. In the simulations presented above, we have changed meridional circulation abruptly after every 30 yrs. To check how the above results change with this coherence time τ, we have done several simulations with different τ. We see that, even when $\tau \sim 15$ yrs, we get around 15–20 grand minima. However, when τ is less than this value, the number of grand minima gets very much reduced. Another important point: instead of changing meridional circulation abruptly at one time, if we change it smoothly in few years, then also the results do not change significantly.

When sunspots are absent, the Babcock-Leighton process for the generation of poloidal field cannot take place. Presumably, during a grand minimum, the poloidal field has to be generated by the α effect originally proposed by Parker [36] and by Steenbeck, Krause, and Rädler [37]. It is possible that an α effect coexists along with the Babcock-Leighton mechanism all the time, although its nature and spatial distribution (even its sign) are completely unclear at this time. In view of this uncertainty, our results presented in Figs. 3 and 4 are obtained by using the same form of α at all times. Once the Sun is pushed into a grand minimum, it comes out of the grand minimum in a time of the order of dynamo growth time, as discussed in detail by Choudhuri and Karak [30]. One intriguing fact to note is that we get durations of grand minima similar to their observed values when we keep using the same α throughout the grand minima.

We conclude that the irregularities of solar cycles, including the grand minima, are produced by fluctuations in

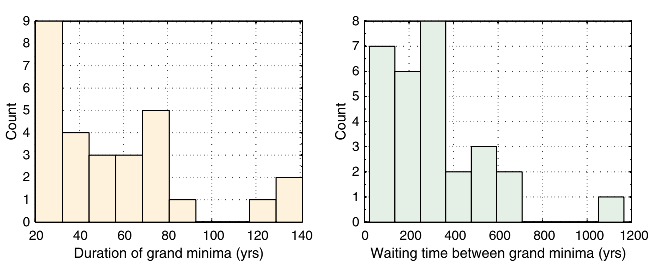

FIG. 5 (color online). The left panel shows the distribution of the durations of the grand minima, and the right panel shows the distribution of the waiting times between the grand minima.

poloidal field generation and in meridional circulation. Assuming these fluctuations to obey Gaussian distributions, we obtain the basic parameters of the Gaussians from the distribution of the peak sunspot numbers and the durations of the last 28 solar cycles. On running our code with such fluctuations, we find that our theoretical dynamo model produces grand minima at a frequency remarkably close to what is found in the data for the last 11 000 yrs.

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