

Related topics

Characteristic X-radiation, energy levels, crystal structures, reciprocal lattices, Miller indices, atomic form factor, structure factor, and Bragg scattering

Principle

The spectra of the X-rays that are reflected with various different orientations by NaCl monocrystals are analysed. The associated interplanar spacings are determined based on the Bragg angles of the characteristic lines.

Equipment

1 XR 4.0 expert unit	09057-99
1 X-ray goniometer	09057-10
1 X-ray plug-in unit with a Cu X-ray tube	09057-50
1 Counter tube, type B	09005-00
1 X-ray universal crystal holder for X-ray units	09058-02
1 X-ray NaCl monocrystals, set of 3	09058-01
1 X-ray diaphragm tube, d = 2 mm	09057-02
1 X-ray diaphragm tube, d = 1 mm	09057-01
1 measure XRM 4.0 X-ray software	14414-61
1 Data cable USB, plug type A/B	14608-00

Additional equipment

PC, Windows® XP or higher

This experiment is included in the upgrade set "XRS 4.0 X-ray structural analysis".



Fig. 1: XR 4.0 expert unit 09057-99

Tasks

1. Determine the intensity of the X-rays that are reflected by the NaCl monocrystals with the orientations [100], [110], and [111] as a function of the Bragg angle.
2. Assign the reflections to the corresponding lattice planes that are given by way of their respective Miller indices.
3. Determine the lattice constant and calculate the interplanar spacing.
4. Determine the mass of a cell and the number of atoms in the cell.

Set-up

Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig. 2). The goniometer block with the analyser crystal should be located at the end position on the right-hand side. Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube (see Fig. 3a). Insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit for the collimation of the X-ray beam.

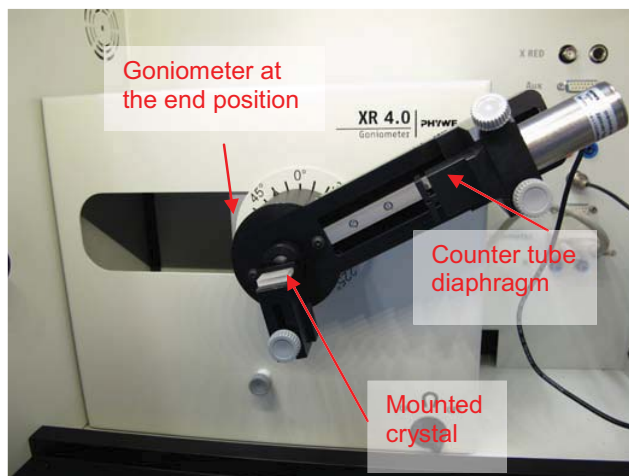


Fig. 3: NaCl crystal in its holder

Procedure

- Connect the X-ray unit via USB cable to the USB port of your computer.
- Start the "measure" program. A virtual X-ray unit will be displayed on the screen.
- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.



Fig. 2: Connectors in the experiment chamber

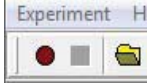
Note

Details concerning the operation of the X-ray unit and goniometer as well as information on how to handle the monocrystals can be found in the respective operating instructions..



Fig. 4: Connection of the computer

- Click the experiment chamber to change the parameters for the experiment. Select the parameters as shown in Figure 6.
- If you click the X-ray tube, you can change the voltage and current of the X-ray tube. Select the following: anode voltage $U_A = 35$ kV; anode current $I_A = 1$ mA.
- Mount one of the crystals in the universal crystal holder and attach it to the goniometer (Fig. 3).
- Start the measurement by clicking the red circle



Note

Never expose the Geiger-Müller counter tube to the primary X-radiation for an extended period of time.

Overview of the settings of the goniometer and X-ray unit:

- 2:1 coupling mode
- Gate time 2 s; angle step width 0.1°
- Scanning range 3° - 60°
- Anode voltage $U_A = 35$ kV; anode current $I_A = 1$ mA

Theory

If X-rays hit a parallel lattice plane family with the interplanar distance d under the glancing angle ϑ , the radiation will be reflected in a constructive manner provided that the so-called Bragg condition (2) is fulfilled (see Figure 7).

$$2d \sin \vartheta = n\lambda; (n = 1, 2, 3, \dots) \quad (1)$$

In the context of crystal structure analyses, n is often integrated into the distance between the lattice planes.

$$2d \sin \vartheta = \lambda \quad (1b)$$

The reflections of the n^{th} order are directly assigned to the diffraction on the various planes.

The Miller indices are a method for naming the various planes in a crystal.

They basically indicate the points of intersection of an imaginary section through the three-dimensional unit cell of the crystal.



Fig. 5: Part of the user interface of the software

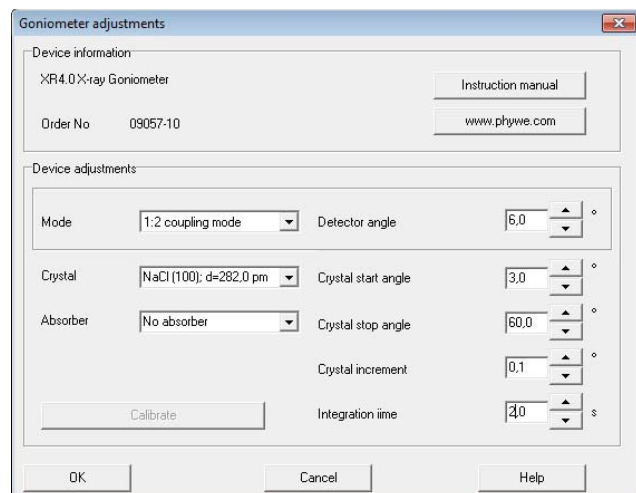


Fig. 6: Settings of the goniometer, NaCl (100) crystal

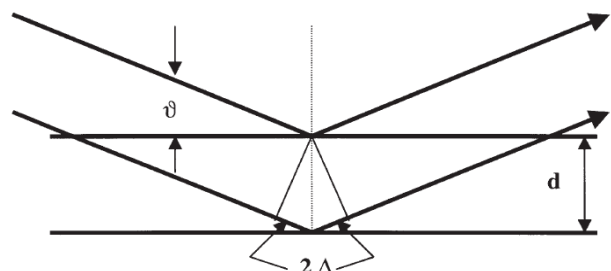


Fig. 7: Bragg scattering on the lattice planes

The symmetrical fundamental unit of a crystal is the unit cell. In a cubic crystal lattice, like in the case of NaCl, all of the sides of this cell are of the same length. The side length of such a cell is referred to as the lattice constant a .

As shown in Figures 8a to 8c, NaCl monocrystals have a face-centred cubic lattice (fcc). In the primitive cell, an Na^+ ion has the coordinates $(0,0,0)$ and a Cl^- ion has the coordinates $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. For a cubic crystal with a lattice constant a , the lattice planes that are characterised by the miller indices (h,k,l) have the following interplanar spacing d :

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (2)$$

Putting (2) into (1b) results in the following connection:

$$\sin \vartheta_{hkl} = \sqrt{h^2 + k^2 + l^2} \cdot \frac{\lambda}{2a} \quad (2b)$$

The relative intensity of the reflected radiation is determined by the scattering power and position of the individual atoms in the unit cell of the crystal. It is described by the so-called structure factor $F(h,k,l)$:

$$F(h,k,l) = \sum_n f_n \cdot \exp[-2\pi i(hu_n + kv_n + lw_n)] \quad (3)$$

In this equation (3), f_n = the atomic form factor (atomic scattering factor), and u_n , v_n , and w_n = the coordinates of the n^{th} atom in the unit cell. The total back-scattered beam intensity I is:

$$I = F^* \cdot F = |F(h,k,l)|^2 \quad (4)$$

With the 000; $0 \frac{1}{2} \frac{1}{2}$; $\frac{1}{2} 0 \frac{1}{2}$; and $\frac{1}{2} \frac{1}{2} 0$ of the basis atoms in the unit cell of an fcc crystal, it follows from (3) that $F = 0$ when the h,k,l triplet contains even and odd numbers, and $F = 4f$ when all of the indices are either even or odd. In addition, in face-centred cubic crystal structures and in the case of the 100 and 110 lattice planes, the reflections of the planes with odd values for h , k , and l are eliminated by systematic extinction. Please refer to the corresponding specialised textbooks.

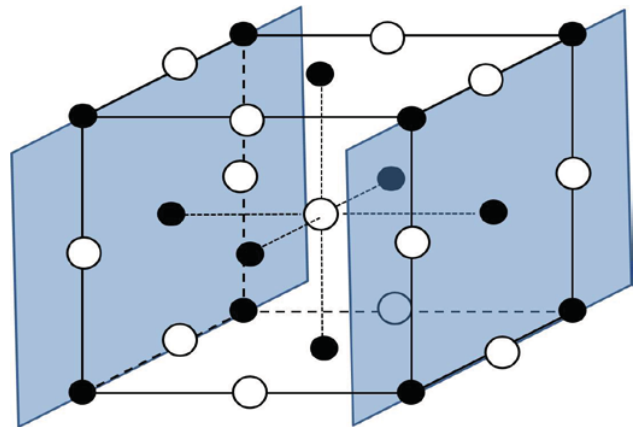


Fig. 8a: NaCl crystal with a drawn-in (100) lattice planes

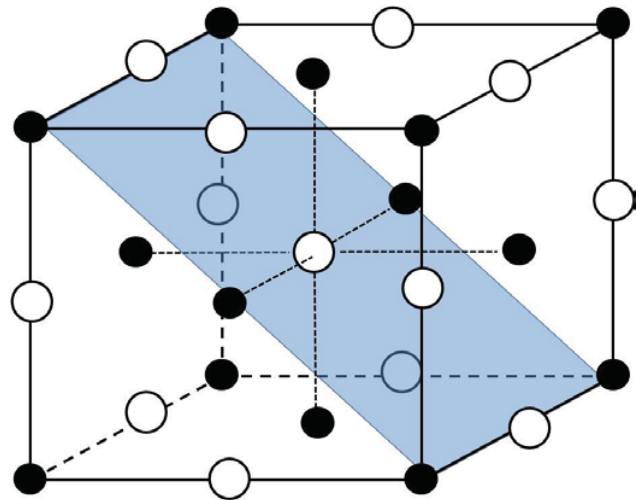


Fig. 8b: NaCl crystal with a drawn-in (110) lattice plane

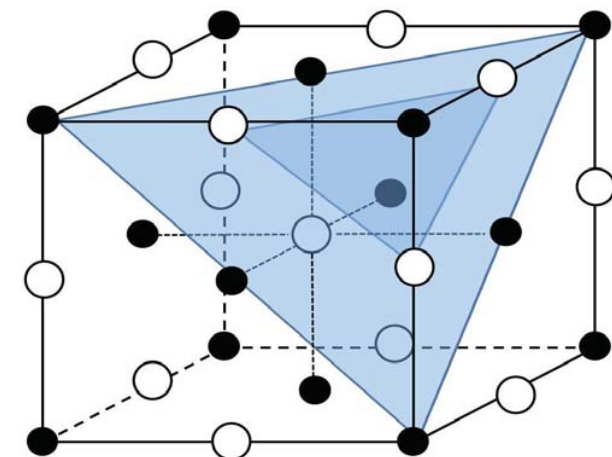


Fig. 8c: NaCl crystal with drawn-in (111) and (222) lattice planes. Na^+ ions = ●; Cl^- ions = ○.

Evaluation

Task 1: Determine the intensity of the X-rays that are reflected by the NaCl monocrystals with the orientations [100], [110], and [111] as a function of the Bragg angle.

Figure 9a to 9c show the spectra of various NaCl crystals.

Compared to the other spectra, the spectrum of the [111] crystal (Fig. 9c) shows a noticeable feature. While in the case of the other spectra, the intensity of the characteristic lines is always the highest for the first order ($n = 1$) reflections. This is the case with $n = 2$ in Figure 9c.

In the [111] crystal, the parallel lattice planes are occupied either only by Na^+ ions or by Cl^- ions. As these two ions have different scattering factors, the intensities also differ from each other. If f_{Na} and f_{Cl} are the scattering factors, the following results from (3) for lattice planes with solely odd (h, k, l) indices:

$$F = 4(f_{\text{Na}} - f_{\text{Cl}}) \text{ and } I \propto F^2 = 16(f_{\text{Na}} - f_{\text{Cl}})^2$$

Accordingly, the following is true for the intensity that is reflected by latticed planes with solely even indices:

$$F = 4(f_{\text{Na}} + f_{\text{Cl}}) \text{ and } I \propto F^2 = 16(f_{\text{Na}} + f_{\text{Cl}})^2$$

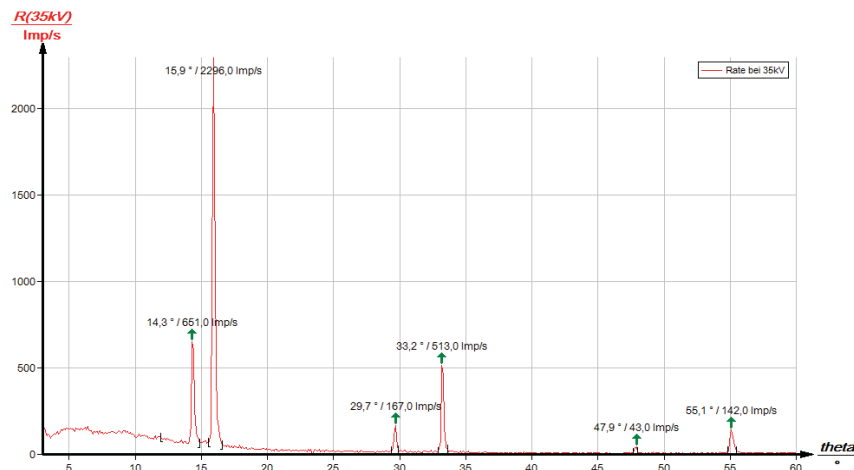


Fig. 9a: Intensity of the X-ray spectrum of copper as a function of the glancing angle ϑ : NaCl monocrystals with different orientations as Bragg analysers: [100] crystal orientation

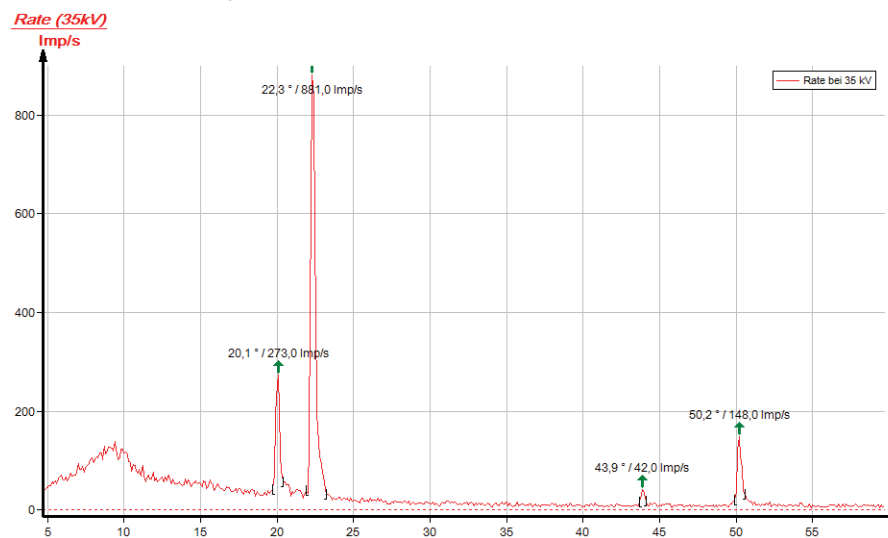


Fig. 9b: [110] orientation

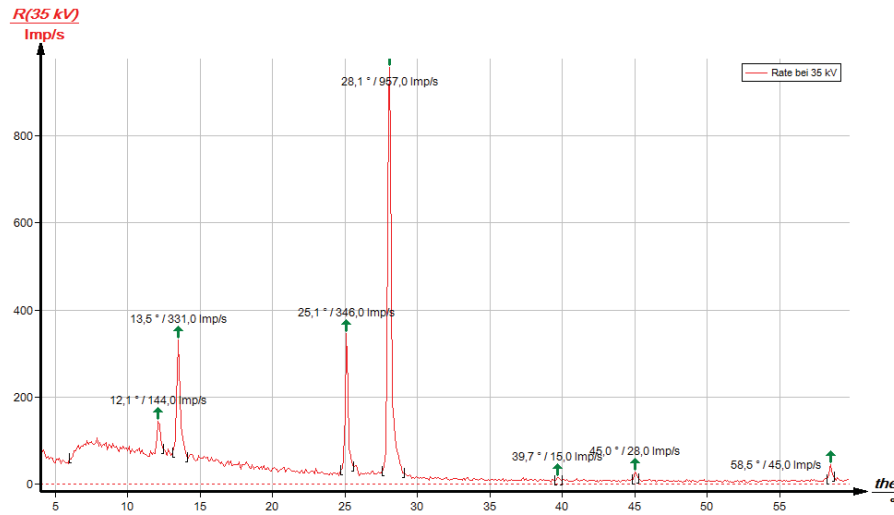


Fig. 9c: [111] orientation

Task 2: Assignment of the Miller indices

In Table 1, the glancing angles ϑ that were determined with the aid of Figures 6a to 6c are assigned to their respective Miller indices. Based on (3), we know that for the 100 and 110 lattice planes only even or only odd values are possible for the h, k, l triplet and that there are in fact no reflections for odd values of h, k , and l at 100 and 110. These considerations resulted in the assignments shown in Table 1.

Table 1

	$\vartheta(K_\alpha)^\circ$	$\vartheta(K_\beta)^\circ$	(hkl)	$(h^2+k^2+l^2)$
(100) crystal				
	15.9	14.3	200	4
	33.2	29.7	400	16
	55.1	47.9	600	36
(110) crystal				
	22.3	20.1	220	8
	50.2	43.9	440	32
(111) crystal				
	13.5	12.1	111	3
	28.1	25.1	222	12
	45.0	39.7	333	27
	-	58.5	444	48

Task 3: Determine the lattice constant and calculate the interplanar spacing.

If one solves equation (2b) for a , one obtains the lattice constant a for the various reflections based on the hkl triplets that were determined in Task 1 as well as based on the glancing angle ϑ and the wavelength of the characteristic X-radiation of copper ($\lambda_{K\alpha} = 154.4$ pm; $\lambda_{K\beta} = 139.2$ pm). Table 2 shows the corresponding values. A comparison of the average value with the literature value of $a = 564$ pm shows good correspondence.

Equation (2) can now be used to calculate the interplanar spacing for the first plane since the lattice constant a refers to a unit cell with only two planes.

With this value and in accordance with equation (1), the distances between the individual lattice planes are as follows: $d(200) = 282.0$ pm, $d(220) = 201.9$ pm, and $d(111) = 330.2$ pm.

Literature values: $d(200) = 282.0$ pm, $d(220) = 199.4$ pm, and $d(111) = 325.6$ pm.

The very good agreement between the value for the distance between the (100) lattice planes that was determined experimentally and the literature value is due to the fact that ionic crystals can be split very exactly in parallel to this plane. The deviations of the other crystals are due to a slight misorientation.

Task 4: Determine the mass of a cell and the number of atoms in the cell.

If one calculates the volume of the unit cell of sodium chloride

$$a^3 = 1.79 \times 10^{-28} \text{ m}^3$$

the density of sodium chloride of $\rho = 2.163 \text{ g/cm}^3$ leads to the weight of a unit cell as

$$m = \rho \cdot V = 2.163 \text{ g/cm}^3 \cdot 1.79 \times 10^{-28} \text{ m}^3$$

$$3.87 \cdot 10^{-25} \text{ kg} = 233 \text{ u}$$

with the atomic mass unit $1 \text{ u} = 1.661 \cdot 10^{-27} \text{ kg}$.

Since the number of Na atoms in NaCl equals the number of Cl atoms, the molar masses $M_{(\text{Na})} = 22.990 \text{ g/mol}$ and $M_{(\text{Cl})} = 35.453 \text{ g/mol}$ lead to a number of $3.99 = 4$ atoms each in the Bravais lattice, which could also be expected for an fcc lattice based on the following considerations (see also Fig. 10):

Every unit cell includes 4 (green) cations:

Each of the cations at the 8 corners of the unit cell counts as only 1/8 to the unit cell since only 1/8 of the volume actually lies within the unit cell.

8 "corner cations" ·
1/8 = 1 cation

The cations on the surfaces count as only 1/2:

6 "surface cations" ·
1/2 = 3 cations

**Resulting total
number: 4 cations**

Every unit cell includes 4 (blue) anions:

Each of the anions at the 12 edges counts as only 1/4 to the unit cell:

12 "edge anions" ·
1/4 = 3 anions

The anion in the centre belongs completely to the unit cell:

1 "centre anion" ·
1/1 = 1 anion

**Resulting total
number: 4 anions**

Table 2

	$\theta/^\circ$	(hkl)	$(h^2+k^2+l^2)$	a/pm
(100) crystal				
K_α	15.9	200	4	567
	33.2	400	16	564
	55.1	600	36	565
K_β	14.3	200	4	564
	29.7	400	16	562
	47.9	600	36	563
		Mean value		564
(110) crystal				
K_α	22.3	220	8	575
	50.2	440	32	568
K_β	20.1	220	8	573
	43.9	440	32	568
		Mean value		571
(111) crystal				
K_α	13.5	111	3	573
	28.1	222	12	586
	45.0	333	27	567
K_β	12.1	111	3	575
	25.1	222	12	568
	39.7	333	27	566
	58.5	444	48	566
		Mean value		572

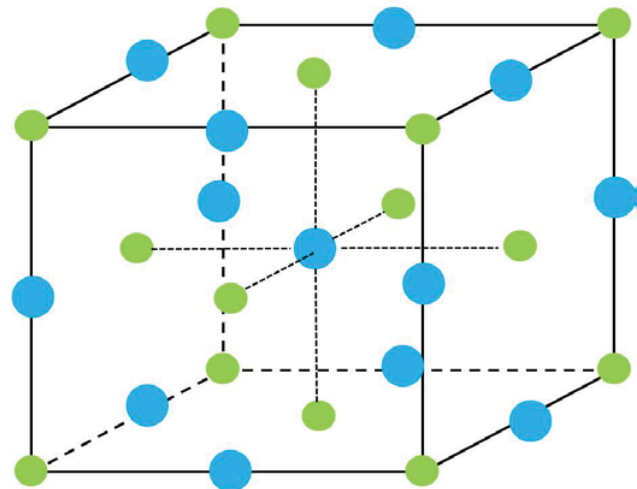


Fig. 10: The unit cell of NaCl, blue: Cl atoms; green: Na atoms