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Wave Phenomena in an Acoustic Resonant Chamber

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A high Q acoustical resonant chamber which can be used to investigate a wide variety of wave phenomena is described. Several short experiments are discussed and typical results are given in graphical form.

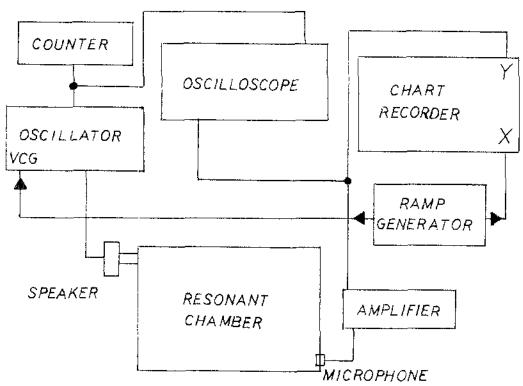

FIG. 1. Schematic diagram of the experimental configuration.

INTRODUCTION

In this article we describe a high Q acoustical resonant chamber which, although conceptually simple and inexpensive to build, can be used to demonstrate a wide variety of wave phenomena. Some of these include normal modes in three dimensions, Q values, the density of states, a simple measurement of the speed of sound, Fourier decomposition, and damped harmonic oscillations. In addition, the chamber has been used in experiments to measure the sound-absorbing properties of various materials and in experiments in which resonant frequencies are shifted by perturbations and scattering caused by small objects placed in the interior of the chamber.

The advantages of using sound and a resonant chamber to study wave phenomena are numerous. Chamber resonances are audible. Wavelengths are such that in the rectangular chamber we describe, whose greatest dimension is two feet, distinct resonances can be observed at frequencies from several hundred hertz to more than a kilohertz, at which point individual resonances become indistinct and contribute to a rising noise level in the chamber. Q values between 100 and 300 are typical for chamber resonances. Finally, because of the relatively low frequencies involved, inexpensive electronics can be used to excite the chamber and monitor its behavior.

The chamber we describe was originally constructed as part of an independent project by a junior physics major interested in architecture and room acoustics. It has subsequently proved useful in demonstrating wave phenomena to students in courses in introductory physics, wave mechanics, mathematical methods of physics, and statistical mechanics.

APPARATUS

The experimental setup is shown schematically in Fig. 1. The chamber is made of $\frac{3}{8}$-in. blackboard slate epoxied together at the edges and painted on the outer surface with epoxy paint to

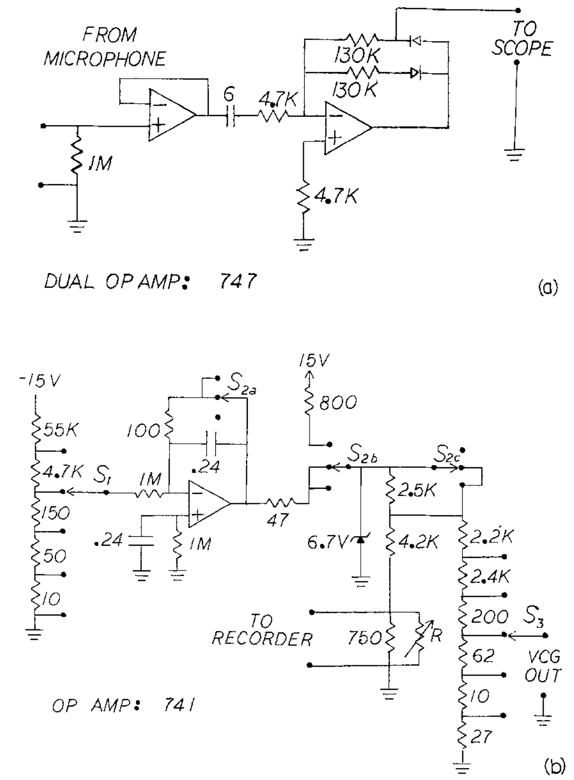

FIG. 2. Circuit diagrams of the half-wave rectifier (a), and ramp generator (b). In the ramp generator circuit, S_2 is a ganged, three-position wafer switch and the adjustable resistor is the fine adjustment for the x–y recorder x-sweep input voltage.

help resist chipping. The interior dimensions, which were chosen to give widely separated resonant frequencies at the low frequency end of the resonance spectrum, are 24 in. $\times$ $16\frac{3}{4}$ in. $\times$ $9\frac{1}{2}$ in. The top is removable and when in place is weighted with small lead blocks to optimize the amplitude of resonances. The entire chamber rests on a platform supported by rubber to minimize vibration.

The chamber is excited by means of a 4-in. loudspeaker enclosed in a small fiberboard box and a 1-$\frac{3}{8}$ in. aluminum tube epoxied in a hole in the upper corner of one side of the chamber. This tube is filled with hollow $\frac{1}{8}$-in. copper tubes to eliminate tube resonances and to reduce the effect of the hole in the chamber wall on its response. A one-inch Shure dynamic microphone cartridge is mounted in a 1-$\frac{3}{8}$ in. hole in the diagonally opposite corner of the chamber with foam rubber packing material and detects sound levels in the chamber. The locations of the sound input and microphone holes are chosen so that the largest possible number of resonant modes of the chamber are excited by the loudspeaker and can be detected by the microphone.[1]

The loudspeaker is driven by a function generator which has a voltage controlled frequency sweep. The oscillator frequency is monitored by a counter. The output of the microphone is amplified and half-wave rectified by the circuit shown in Fig. 2(a). A desirable feature of this circuit is that half-wave rectification occurs without any loss of signal resulting from junction drops in the diodes. The rectified output is then monitored with an oscilloscope and fed into the vertical amplifier of an x–y recorder. The horizontal amplifier of the recorder is driven by the ramp generator circuit shown in Fig. 2(b). This circuit also drives the voltage-controlled frequency sweep of the signal generator.

EXPERIMENTS

Many short experiments can easily be performed with this chamber instrumented in the

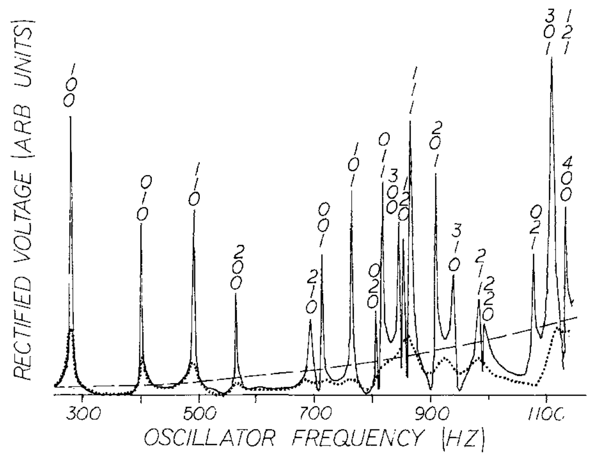

FIG. 3. The chamber resonance spectrum between 250 and 1150 Hz. Mode numbers identify resonant peaks. The dotted line shows the effect on the resonance spectrum of 1-in. fiberglass wool lining in the chamber bottom. The dashed line is related to a theoretical expression for the number of resonances $N(f)$ in the frequency interval 0 to f.

manner described in the last section. In this section we describe several of these experiments, some of which are suitable for lecture demonstrations. Typical results of these experiments are presented in graphical form.

A. Normal Modes and Resonance Q's

A plot of microphone voltage as a function of the frequency of a sinusoidal waveform used to drive the loudspeaker over a range in frequency from 250 to 1150 Hz is shown in Fig. 3. Resonant frequencies can be measured with a counter and also calculated from the equation

$$f=\tfrac{1}{2}c[(n_x/L_x)^2+(n_y/L_y)^2+(n_z/L_z)^2]^{1/2}, \quad (1)$$

where c is the speed of sound, n_x, n_y, and n_z are zero or positive integers, and L_x, L_y, and L_z are the interior dimensions of the chamber with $L_x > L_y > L_z$. When a temperature-corrected value of c is used,[2] resonant frequencies calculated from Eq. (1) typically agree to 0.1% with the measured values.

In Fig. 4 the shape of the lowest frequency (1, 0, 0) resonance is shown on a greatly expanded frequency scale. The Q of this resonance ($f_0=281.2$ Hz) calculated from the equation

$$Q=f_0/\Delta f, \quad (2)$$

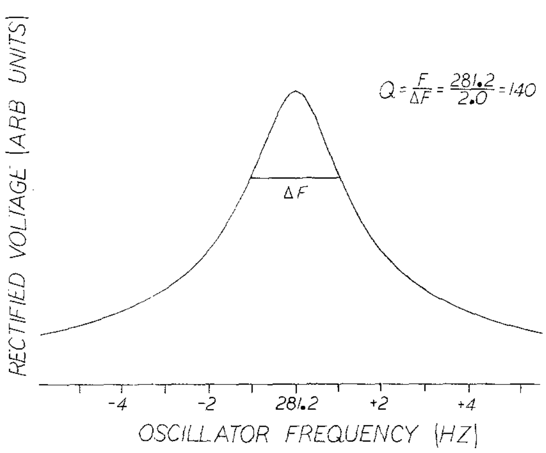

FIG. 4. The (1, 0, 0) resonance on an expanded frequency scale.

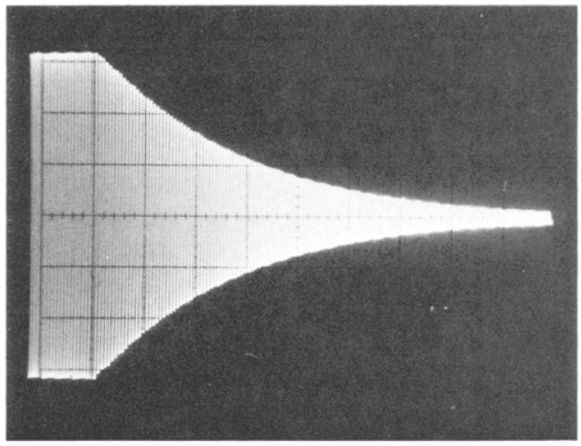

FIG. 5. Oscilloscope trace showing the decay of the (1, 0, 0) resonance. The horizontal scale is .056 sec/div.

where $\Delta f=2.0$ Hz, is the full width of the curve at $1/\sqrt{2}$ of its maximum amplitude, is about 140. Q values for other resonant modes are comparable.

Q values may also be determined by measuring the decay (or rise) time of the resonant oscillations when the signal generator, operating at a resonant frequency, is gated.[3] An oscilloscope trace showing the decay of the (1, 0, 0) mode is shown in Fig. 5. Q values obtained using this method agree with those calculated from line widths.

For frequencies greater than about 1 kHz the number of resonant frequencies increases rapidly so that individual resonances are partially smeared out and begin to blend into a generally increasing background level in the chamber. An approximate expression for the number of resonances $N(f)$ for frequencies between 0 and f is derived by Morse and Ingard[4] and is given by the equation

$$N(f)=(4\pi V/3c^3)f^3+(\pi A/4c^2)f^2+(L/8c)f, \quad (3)$$

where V is the volume of the chamber, A is the total surface area of the interior of the chamber, and L equals $4(L_x+L_y+L_z)$. A plot of Eq. (3) multiplied by an arbitrary constant voltage per resonance is shown by the dashed line in Fig. 3. In the frequency range $0 \leq f \leq 1.1$ kHz, Eq. (3) predicts 20 resonances whereas the actual number, including the two resonances near 1110 Hz is 19. By differentiating Eq. (3) and taking the

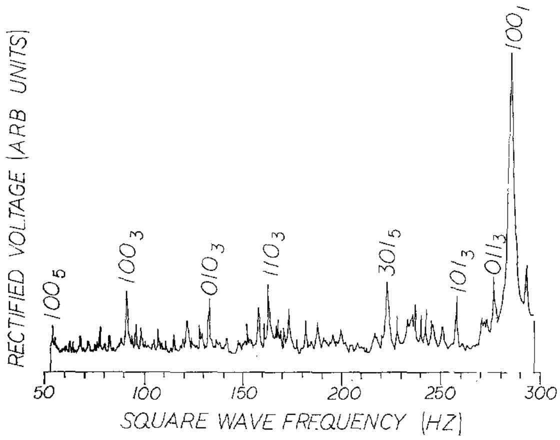

FIG. 6. Resonance peaks when the chamber is excited with a variable frequency square wave. Subscripts on the mode numbers refer to the resonating harmonic of the square wave frequency.

limit $f \to \infty$, the classical equation for the density of states per unit frequency interval is obtained.

When the bottom of the chamber is lined with 1 in. fiberglass wool the effect on the resonance spectrum in the chamber is shown by the dotted line in Fig. 3. The fiberglass causes a reduction in the amplitudes of each resonant mode by coupling normal modes to produce a standing wave with randomly distributed nodal surfaces. An extensive mathematical analysis of the effects of sound-absorbing walls on noise levels in rectangular chambers is given in Chap. 9 of Ref. 4.

Finally, it should be noted that when the chamber resonates in its lowest mode, the speed of sound can be calculated trivially from the equation

$$c = 2lf_0, \tag{4}$$

where l is the long dimension of the chamber. Because of the high Q of the chamber and the ease of measuring frequency, c can be found this way with a precision of 0.1%.

B. Fourier Decomposition

The high Q of the cavity makes it a selective frequency filter. For example, when the loudspeaker is driven with a square wave whose frequency is varied continuously from 300 Hz down to 50 Hz, resonance peaks in the frequency spectrum are seen when the square-wave frequency $f_{\rm SW}$ satisfies the relation

$$f_{\rm SW} = f_{\rm CR}/n, \tag{5}$$

where $f_{\rm CR}$ is the frequency of a chamber resonance and n is an odd positive integer. This spectrum is shown in Fig. 6. The frequency scale is the square-wave frequency $f_{\rm SW}$, and several high-amplitude peaks are identified as harmonics of resonant modes of the chamber. Peaks corresponding to the third and fifth harmonics of the square-wave frequency f_0 are observed in this frequency range since they cause the chamber to resonate in its lowest (1, 0, 0) resonant mode. Other peaks result from harmonics of square wave frequencies which cause the chamber to resonate in higher frequency resonant modes.

C. Boundary Perturbations and Scattering

When objects whose dimensions are small compared to the chamber dimensions are placed along the bottom surface of the chamber interior, the resonance frequencies are slightly perturbed. When the object is rectangular in shape and oriented so that its surfaces are parallel to the chamber walls, this perturbation can be calculated from boundary perturbation theory described by Morse and Feshbach.[5] The first order shift in frequency Δf for the lowest resonance frequency f_0 calculated from this theory is given by the equation

$$\frac{\Delta f}{f_0} = -\frac{\sum_i[\sin(2\pi/L_x)x]\eta_i S_i}{\sum_i\{(2\pi/L_x)x+\sin[(2\pi/L_x)x]\}\eta_i S_i}, \tag{6}$$

where the sum runs over all surfaces parallel to the yz plane, S_i is the area of the ith surface, η_i has the value $+1$ when the outward normal of the boundary surface points in the positive x direction and -1 when the outward normal points in the negative x direction, and it is assumed that x is the long dimension of the chamber.

Equation (6) predicts that objects located near the center of the chamber will reduce the resonance frequency whereas objects located near the ends of the chamber will increase it. These predictions are easy to check and are in good quantitative agreement with the experiment when the perturbing object is small enough so the shift in the resonance frequency is less than 1 Hz. Calculated and measured frequency shifts for a rectangular strip of steel $9\frac{1}{2}$ in $\times$ $1\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. placed on the bottom surface of the chamber at various positions from the chamber walls are shown in Fig. 7.

When a thin rectangular strip is mounted between the side walls of the chamber away from the floor and the ceiling, some of the resonance frequencies are shifted slightly due to the scattering of waves by the strip. A complete analysis of this problem is also given by Morse and Feshbach.[6] The expression for the resonant frequency when a thin strip is oriented so that its faces are parallel to the yz plane and its centerline is a distance y_0 from the bottom surface of the chamber and a distance x_0 from one of the end walls is

$$f=\frac{c}{2\pi}\left\{\left(\frac{n_x\pi}{L_x}\right)^2\left[1-\frac{\pi\epsilon_x\epsilon_y}{4}\left(\frac{a}{L_x}\right)^2\times\cos^2\left(\frac{n_y\pi y_0}{L_y}\right)\sin^2\left(\frac{n_x\pi x_0}{L_x}\right)\right]+\left(\frac{n_y\pi}{L_y}\right)^2+\left(\frac{n_z\pi}{L_z}\right)^2\right\}^{1/2}, \quad (7)$$

where $\epsilon_x=1$ if $n_x=0$ and 2 if $n_x\geq 1$, $\epsilon_y=1$ if $n_y=0$ and 2 if $n_y\geq 1$, and a is the width of the strip.

Plots of calculated and experimental frequency shifts from the resonance frequencies of the (1, 0, 0), (2, 0, 0), and (3, 0, 0) modes are given in Fig. 8 for an aluminum strip $\frac{1}{16}$-in. thick and $1\frac{1}{2}$-in. wide centrally mounted between the chamber side walls with adhesive tape. It can be seen from the figure that Eq. (7) is in good qualitative agreement with the experimental results.

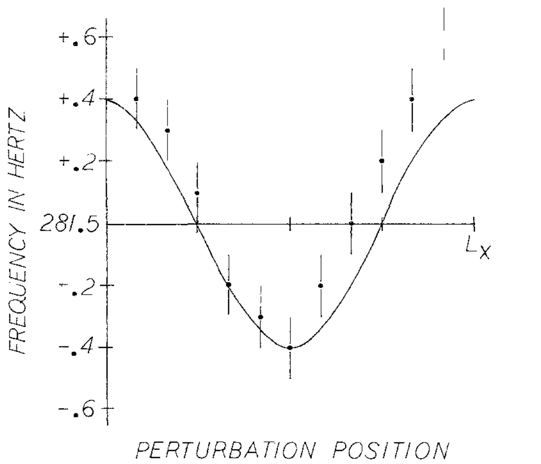

FIG. 7. Perturbations in the frequency of the (1, 0, 0) mode due to a small rectangular boundary perturbation in the floor of the chamber. The solid line is calculated from Eq. (6) in the text.

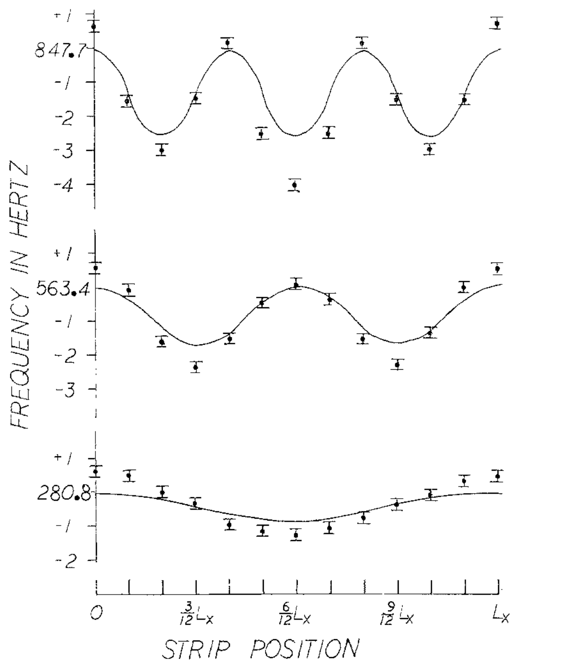

FIG. 8. Frequency shifts of the (1, 0, 0), (2, 0, 0), and (3, 0, 0) modes due to scattering from a thin aluminum strip mounted between the side walls of the chamber. Solid lines are calculated from Eq. (7) in the text.

CONCLUSIONS AND ACKNOWLEDGMENTS

Although a complete study of acoustics requires elaborate experimental apparatus and mathematical sophistication beyond the level normally achieved by undergraduate physics majors, many basic wave phenomena can be demonstrated in a conceptually straightforward way by means of a simple acoustical resonant chamber. The resonance peaks of the particular chamber we describe are quite spectacular and are narrow enough to impress students with the meaning of the terms resonance and resonance Q. Students also readily appreciate the usefulness of the concept of normal modes and can see the increase of the density of modes per unit frequency interval with increasing frequency. Other experiments we describe can equally well be demonstrated with other apparatus, but it is satisfying for students to see that well known wave principles can be used to understand the data obtained from the chamber in these experiments.

It should be emphasized, however, that the use of such a chamber is not limited to the rather short demonstration-type experiments described in the last section. The high Q of the chamber enables resonant frequencies to be easily measured to a tenth of a hertz, making it possible to study changes in the speed of sound with changes in temperature and changes in humidity. A number of problems in perturbation theory and scattering theory can be studied experimentally with the chamber. In addition, a wide variety of problems in the physics of music and architectural acoustics can be investigated with the chamber in a quantitative or semiquantitative way. Other experiments will no doubt suggest themselves to the reader.

The performance of a resonance chamber of this type depends entirely on the care with which it is constructed. We are indebted to Charles Lang for his craftsmanship and ingenuity in building the chamber.

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[1] Many of the ideas for the chamber design came from an article by N. B. Bhatt, J. Acoust. Soc. Am. **11**, 69 (1940).

[2] Equations for the variation of c with temperature are given in L. L. Beranek, *Acoustic Measurements* (Wiley, New York, 1949), p. 47.

[3] The relation between Q and τ, the time constant of the decay, is given by the equation $Q=\pi f\tau$ where f is the frequency of the resonance. See J. Hunter, *Acoustics* (Prentice Hall, Englewood Cliffs, NJ, 1957), p. 13.

[4] P. M. Morse and K. U. Ingard, *Theoretical Acoustics* (McGraw-Hill, New York, 1968), pp. 585–587.

[5] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics, Part II* (McGraw-Hill, New York, 1953), pp. 1038–1064, 1166.

[6] See Ref. 5, p. 1439.