Intermediate Electromagnetism, UP 203

Problem Set III

1. Two dipoles $\vec{p}_1$ and $\vec{p}_2$ are placed far away from each other. If $\vec{r}$ is the separation between them, show that the interaction energy between the dipoles is given by

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})\right].$$

   Also calculate the force exerted by the dipoles on each other. What is the minimum energy configuration for the two dipoles separated by a distance r?

2. A dipole is situated a distance z above an infinite grounded conducting plane. The dipole makes an angle θ with the perpendicular to the plane. Find the torque on $\vec{p}$. If the dipole is free to rotate, in what orientation will it come to rest? (from Griffiths)

3. Four charges are placed in the $y-z$ plane: charge q at $(0, 0, -a)$, $3q$ at $(0, 0, a)$, $-2q$ at $(0, -a, 0)$, and another $-2q$ at $(0, a, 0)$. Find an approximate formula for the potential, valid far from the origin; go only till $\mathcal{O}(r^{-3})$ terms in the expansion. Express your answer in spherical coordinates.

4. Calculate the monopole, dipole, and quadrupole moments for the following charge distribution: charge q at $(0, 0, a)$, charge q at $(0, 0, -a)$, and charge $-2q$ at the origin. How does the leading order term in the potential go with r for distance $r \gg a$?

5. Two linear dielectrics of permittivity ϵ_1 (thickness d_1) and ϵ_2 (thickness d_2) are placed within a parallel plate capacitor of thickness with (free) charge surface density $\pm\sigma$. Calculate the electric displacement $\vec{D}$, electric field $\vec{E}$, polarization $\vec{P}$, and bound charge densities everywhere within the capacitor.

6. A linear dielectric sphere of radius a is placed in an external constant field $\vec{E}_0$. The field is modified due to the placement of the dielectric. Find the electric potential everywhere in the new configuration. Calculate the induced surface charge density on the dielectric sphere. Compare this with the case of a perfect conductor.

7. **Method of images for dielectrics:** A point charge q is placed at a distance d from a semi-infinite slab of a linear dielectric material with permittivity ϵ. Find the electric potential everywhere and the surface charge density induced on the dielectric. Hint: Try for two image charges q', q'' at d and $-d$ from the surface of the dielectric and adjust them to satisfy the jump conditions at the surface of the dielectric. (see the first part of section 4.4 in Jackson)

8. The electric field in a volume V in vacuum is given by $\vec{E}_0$. With the sources of this electric field fixed in position, a linear dielectric material of volume V_1 is introduced inside V. Show that the change in the energy is

$$W = -\frac{1}{2}\int_{V_1} \vec{P} \cdot \vec{E}_0 d^3x,$$

   where $\vec{P}$ is the polarization in the dielectric. What is the interpretation for the change in the energy? Recall that polarization is nothing but the average dipole moment per unit volume. (see section 4.7 in Jackson)