Intermediate Electromagnetism, UP 203

Problem Set IV

1. Prove the following by generalizing the divergence and Stokes' theorems:

$$\int_V d^3r\vec{\nabla} f = \oint_S d\vec{S} f; \int_V d^3r\vec{\nabla}\times\vec{A} = \oint_S d\vec{S}\times\vec{A}$$

$$\int_S d\vec{S}\times\vec{\nabla} f = \oint_C d\vec{l} f; \int_S (d\vec{S}\times\nabla)\times\vec{A} = \oint_C d\vec{l}\times\vec{A}.$$

2. Prove the following for magnetostatics: (a) $\int_V \vec{J}(\vec{r}')d^3r' = 0$;
(b) $\int_V (\vec{r}\cdot\vec{r}')\vec{J}(\vec{r}')d^3r' = -(1/2)\vec{r}\times\int_V \vec{r}'\times\vec{J}(\vec{r}')d^3r'$, where V is the volume enclosing the current distribution.

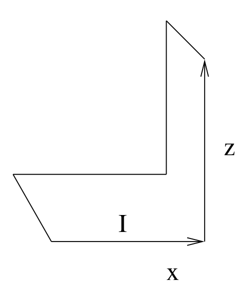
3. Using the dipole vector potential $\vec{A} = \mu_0(\vec{m}\times\vec{r})/4\pi r^3$, show that the magnetic field at a large distance from a magnetic dipole $\vec{m}$ is given by

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3\hat{r}(\hat{r}\cdot\vec{m}) - \vec{m}}{r^3}\right],$$

where $\vec{m}$ is the magnetic dipole moment. The earth's magnetic field can be well-approximated as a dipole. The approximate field strength on earth's surface at the magnetic equator is $B_0 \simeq 3\times 10^{-5}$ T. Find the expression for the field components B_r and B_θ at a distance $r(> R)$ from earth's center and at an angle θ from the magnetic north pole. An electron at $2R_E$ (twice the earth radii) on magnetic north pole is executing a uniform circular motion with a radius (known as Larmor radius) of 1 m. What will be the Larmor radius of a proton with the same energy at $3R_E$ on the magnetic equator? What is the direction of rotation for our electron and proton? *Lesson:* Earth's magnetosphere protects us from energetic particles coming from the sun by trapping these particles, directing some of them to the poles which cause beautiful auroras.

4. Suppose that the magnetic field in some region is given by $\vec{B} = kz\hat{x}$ (where k is a constant). Find the current distribution which is responsible for this magnetic field. Find the force on a square loop (side a) , lying in the $y - z$ plane and centered at the origin, if it carries current I, flowing counter-clockwise, when you look down the $x-$ axis. (adapted from Griffiths)

5. (a) Find the magnetic field at the center of a square loop of side $2R$ which carries a steady current I. (b) Find the field at the center of a regular $n-$sided polygon carrying a steady current I (again the distance of the center to the sides is R). (c) Check that the formula reduces to the field at the center of a circular loop in the limit $n\to\infty$. (from Griffiths)



6. Find the magnetic dipole moment for the current loop in the figure; each side is of length a. Also, find the vector potential and magnetic field vector far away at $\vec{r}(x, y, z)$.

7. Two infinite parallel plates charged with a charge density $\pm\sigma$ are moving with the same velocity v along the plates. (a) Find the magnetic field between the plates, and also above and below them. (b) Find the magnetic force per unit area on the upper plate, including its direction. (c) At what speed v would the magnetic force balance the electric force? (from Griffiths)

8. What current density would produce the vector potential $\vec{A} = k\hat{\phi}$ (where k is a constant) in cylindrical coordinates. Also, find the vector potential $\vec{A}$ for a long current carrying wire. (from Griffiths)

9. We derived the jump conditions for the magnetic field across a boundary with the surface current density $\vec{K}$. Using similar arguments prove that the vector potential $\vec{A}$ is continuous across the boundary. Also show that,
$$\frac{\partial \vec{A}_{\text{above}}}{\partial n} - \frac{\partial \vec{A}_{\text{below}}}{\partial n} = -\mu_0 \vec{K}.$$
This is analogous to to the jump condition for the normal derivative of the electrostatic potential in electrostatics.