Intermediate Electromagnetism, UP 203

Problem Set V

Most problems below are adapted from Griffiths.

1. Prove that free charges are diamagnetic in nature; i.e., the induced magnetic dipoles point opposite to the direction of the magnetic field. Using the formula for the force on a magnetic dipole in an external field, $\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$, show that both electrons and protons feel a force that points in the direction of the decreasing field strength. *Note that for a charge particle in circular orbit in a magnetic field the angular momentum (or dipole moment) is roughly invariant if the Larmor radius is much smaller than the scale over which field strength changes.*

2. A short circular cylinder of radius a and length L carries a frozen-in uniform magnetization $\vec{M}$ parallel to its axis. Find the bound current, and sketch the magnetic field of the cylinder everywhere. Write the expression for the field strength at large distances ($r \gg L$).

3. An infinite solenoid (with n turns per unit length, current I) is filled with a linear material of susceptibility χ_m. Find the magnetic field inside and outside the solenoid. Find the self-inductance of the solenoid; self-inductance is defined as the ratio of the magnetic flux and the (free) current. Compare with the case in vacuum.

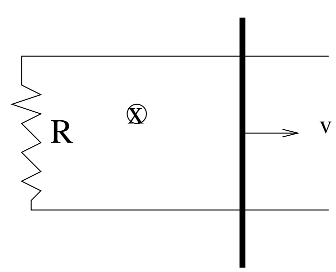
4. A sphere of linear magnetic material is placed in an otherwise uniform magnetic field $\vec{B}_0$. Find the field inside and outside the sphere.

5. At the interface between two linear magnetic materials show that $\tan\theta_2/\tan\theta_1 = \mu_2/\mu_1$, where $\theta_{1,2}$ is the angle between $\vec{B}_{1,2}$ and the normal; this is equivalent to Snell's law for pure magnetic fields. Assume that there is no free current at the interface.

6. Two long cylinders (radii a and b) are separated by a material of conductivity (σ). If the two cylinders are maintained at a potential difference V, what current flows from one cylinder to the other in a length L. Assume that the electric field in the material satisfies Ohm's law ($\vec{J} = \sigma\vec{E}$). Calculate the resistance of this circuit.

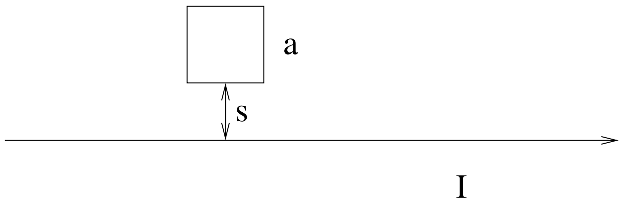
7. A charged capacitor with charge $\pm Q$ is connected through a resistor R in a circuit at $t = 0$. How do charge on the capacitor and current in the circuit evolve with time? Where does the energy dissipated in the resistor come from?

8. A battery with emf $\mathcal{E}$ and internal resistance r is hooked up to a variable resistance R. For what value of R will the power delivered to the load be maximum?

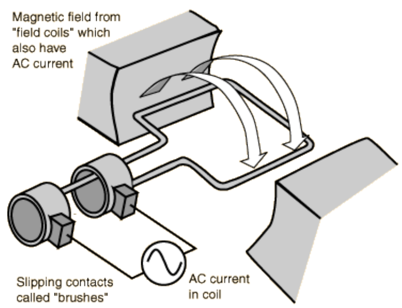


9. A metal bar of mass m slides frictionlessly on two parallel conducting rails distance l apart. A resistor R is connected across rails and a uniform magnetic field pierces the circuit (see the figure above). If the bar moves to right at speed v, what is the current in the resistor? In what direction does it flow? What is the magnetic force on the bar? In what direction? If the bar

starts out at v_0 at $t = 0$, and is left to slide, what is the speed at a later time t? The initial kinetic energy of the bar is $mv_0^2/2$. Check that the energy delivered to the resistor by the time the bar comes to rest is the same.



10. A square loop of wire (side a) lies at a distance s from a very long straight wire carrying current I as shown in the figure above. Find the magnetic flux through the loop. If the loop is pulled away from the wire at velocity v, what is the emf generated? In what direction does the current flow? What if the loop is pulled to the right at speed v?



11. A square loop of side a is mounted on a shaft and rotated at angular velocity ω in a uniform magnetic field B (see above figure). Find $\mathcal{E}(t)$ for this AC generator. In an AC motor an AC current is driven in the square loop and Lorentz forces produce torque which leads to rotation.

12. A long solenoid of radius a is driven by an AC current so that the field inside the solenoid is $\vec{B} = B_0 \cos(\omega t)\hat{z}$. A circular loop of radius $a/2$ and resistance R is placed inside the solenoid, and coaxial with it. Find the current induced in the loop as a function of time.

13. A long coaxial cable carries current I; the inner and outer cylinders (of radii $a,\ b$) carry currents in opposite directions. Find the magnetic energy stored in a section of length l.

14. Suppose
$$\vec{E}(\vec{r}, t) = -\frac{q}{4\pi\epsilon_0 r^2}\theta(vt - r)\hat{r};\ \vec{B}(\vec{r}, t) = 0$$
(θ function is the step function). Show that these fields satisfy Maxwell's equations, and determine ρ and $\vec{J}$. Describe the physical situation that gives rise to these fields.