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What Kind of Topological Phases can be Found in Fractals?

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Overview

- Brief review of non-interacting topological phases
- Question: Topological phases on fractals
- Model and results

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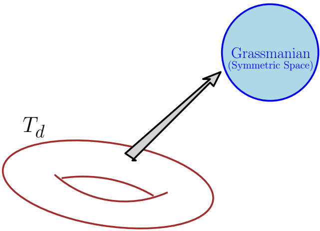
- Homogeneous fractals *do not* host gapped topological phases...
- ...a new gapless phase dubbed as ***fractalized metal***

Topological Insulators : Quick Review

- Early clues: quantum hall effect
- Current understanding based on band theory of periodic lattices

Cartan\\d	0	1	2	3	4	5	6	7	8	
Complex case:										
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
Real case:										
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\cdots$
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$\cdots$
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

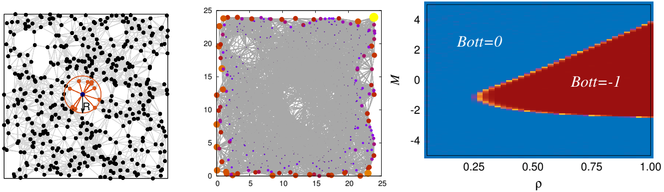


(Kitaev (2009), Ryu et. al. (2010), Ludwig:1512:08882)

- Ground state described by a set of filled band, can be viewed as a map from the d-dimensional Brillouin zone ($\equiv T_d$, d-torus) a Grassmanian manifold
- Topological phases exist if such maps can "twist and wind" (as characterized by the nontrivial groups)

Key Issue

- Topological phases: gapless edge states robust to disorder
- Quasicrystalline systems (Kraus et al., *PRL* (2012), Tran et al., *PRB* (2015), Fulga et al., *PRL* (2016), Bandres et al., *PRX* (2016)) can host topological phases
- ...and even amorphous systems (Agarwala and VBS, 1701.00374 o $\Longrightarrow$ *PRL* (2017))



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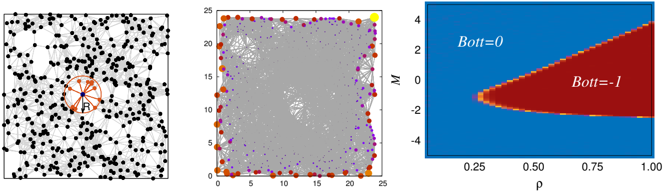

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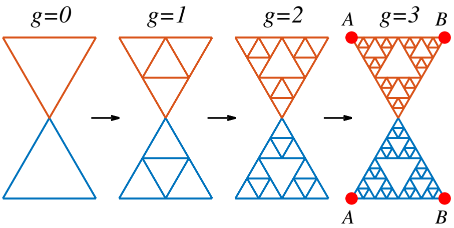
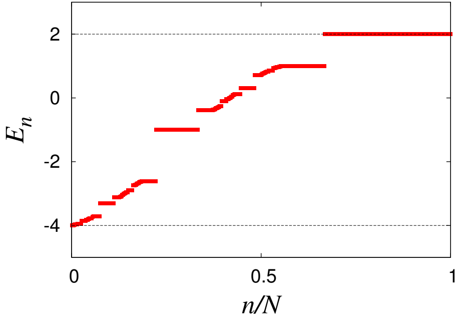
- Key point: Only the symmetry class and spatial dimension is crucial; related notion of *bulk* and *edge*

Question

- What about systems where the notion of bulk and edge is not sharp? Example: fractal lattice
- Topological phases possible in fractal lattices?

Fractal Lattices





- Sierpinski gasket – our workhorse – Housdorff dimension = $\log 3/\log 2$
- Build generation by generation starting from a triangular motif
- Identify A and B sites to realize a homogeneous fractal – all sites are equally coordinated
- Number of sites N in generation g: $N = 3^{g+1}$
- Notion of bulk and edge is not sharp – operating definition: sites of the latest generation are the bulk sites leads to $N_e/N_b = 1/2$
- Simple tight binding model (Domany et al., *PRB* (1983)) leads to a self similar spectrum with infinite number of band gaps in the thermodynamic limit

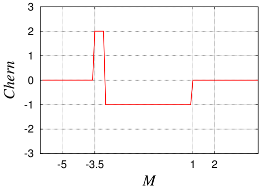
“Topological Hamiltonian” on Fractals

- Two orbital model inspired by BHZ (Bernevig, et al., *Science* (2006))

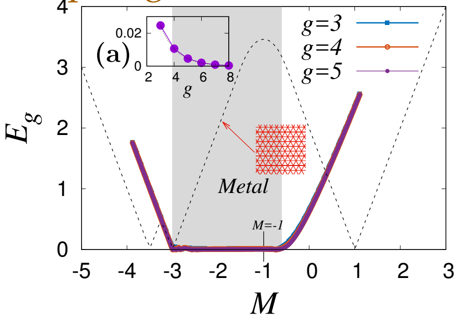
$$\mathcal{H} = \sum_{I\alpha} \sum_{J\beta} t_{\alpha\beta}(\boldsymbol{r}_{IJ}) c^{\dagger}_{I,\alpha} c_{J,\beta}$$

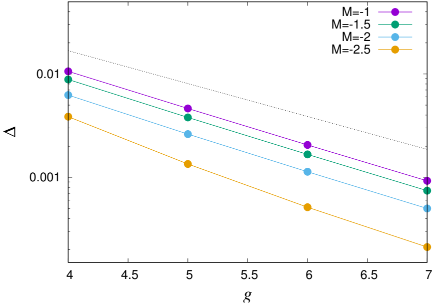
$$t_{\alpha\beta}(\boldsymbol{r} = \boldsymbol{0}) = \mathrm{Diag}\{2 + M, -(2 + M)\}, \quad t_{\alpha\beta}(\boldsymbol{r} \neq \boldsymbol{0}) = \begin{pmatrix} \frac{-1}{2} & \frac{-ie^{-i\theta}}{2} \\ \frac{-ie^{i\theta}}{2} & \frac{1}{2} \end{pmatrix}$$

  θ is the angle made by the bond with the x-axis
- Half-filling – one fermion per site
- M is the mass parameter which can be tuned to change the topology
- On triangular lattice topological phases are realized for $-3.5 \leq M \leq 1$



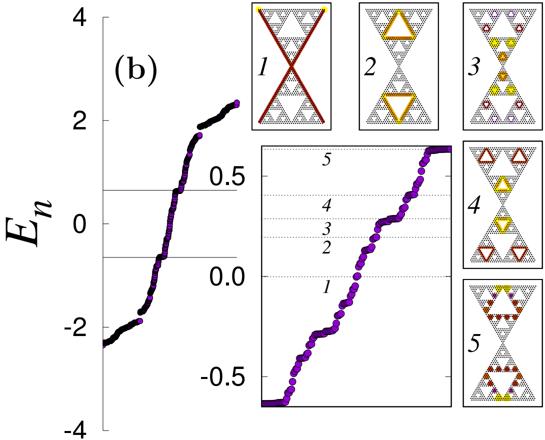
"Topological Hamiltonian" on Fractals – Ground State

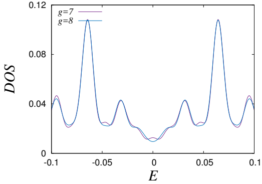


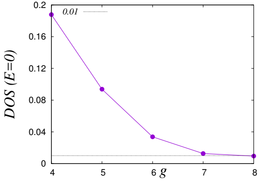


- When M is in the "topological regime" $-3 \leq M \leq 0$, no gapped phase is found on the fractal
- For any M in this regime, energy gap goes to zero exponentially in increasing generation – system becomes gapless!

Nature of the Gapless State

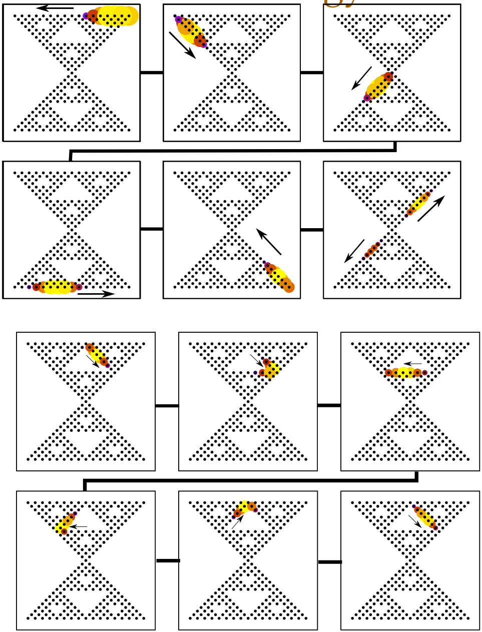






- Low energy states "live" on "edges of different generations" with distinct spatial structure imbibed from the fractal
- In the thermodynamic limit there are low energy states live edges of "all" generations – **fractalized metal**
- These these give a finite density of states at the chemical potential

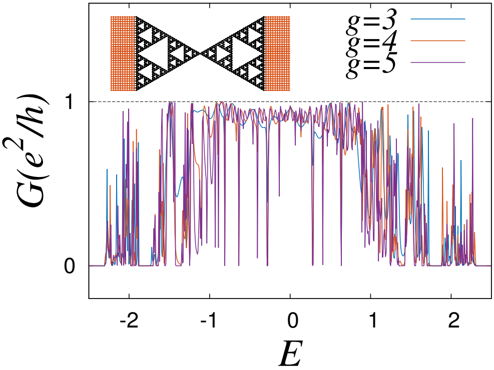
Motion of "Low Energy" Wave Packets



- Wave packets localized near "edges" of different generations comprised of low energy states have distinct "chiral" motion – quite different from a usual metal

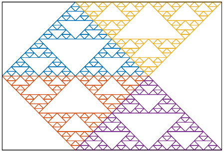
Transport Properties of the Fractalized Metal

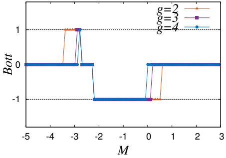
- Fractalized metal is topologically trivial (Bott index = 0)
- Two terminal transport is "nearly quantized", but not quite!

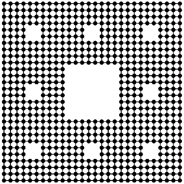


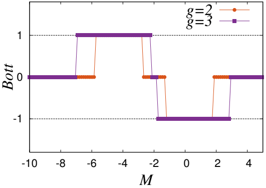
- Dips and fall arise from the fractalized nature...not all low energy states hybridize with the lead states – very different from a usual metal
- Robust to Anderson on-site disorder

Do All Fractals Support Fractalized Metals?



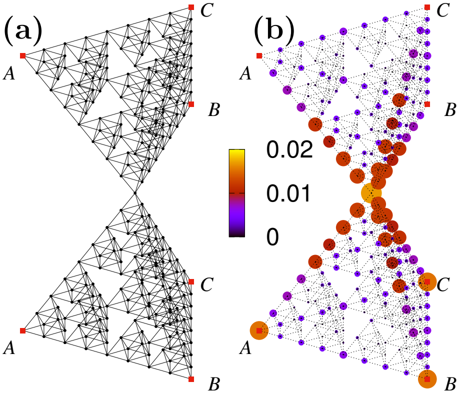






- No!
- One can construct fractal like system which are inhomogeneous – not every site is similarly coordinated – the notion of bulk and edge becomes sharper in these systems and one obtains fully gapped topological phases
- Suggests a necessary condition for fractalized metal – homogeneous fractal!

“Higher Dimensional” Fractalized Metals



- Higher dimensional fractalized metals are also possible!

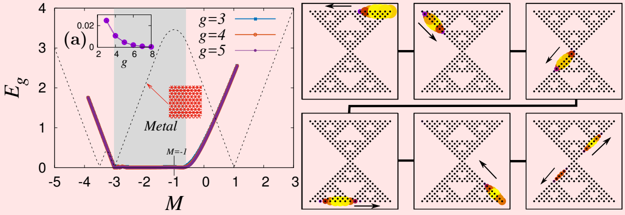
Summary

Question

- What kind of topological phases can be found in fractals?

Answer (1803.01404)

- Inhomogeneous fractals (where the definition of bulk and edge is sharper) may host usual gapped topological phases
- Key new finding: Homogeneous fractals do not support gapped topological phases



- A new *fractalized metal* is realized

Further work: properties of fractalized metals, effects of interaction etc.

Robustness to Disorder