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Strange Half-Metals and Mott Insulators in SYK Models

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Based on arXiv:1703.05111

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- Key contributor:

Arijit Haldar

Overview

- Review and motivation
  - Quick recap of half-metals and Mott insulators
- Our model: Two SYK dots coupled via *interactions*
- Analysis and results

Overview

- Review and motivation
  - Quick recap of half-metals and Mott insulators
- Our model: Two SYK dots coupled via *interactions*
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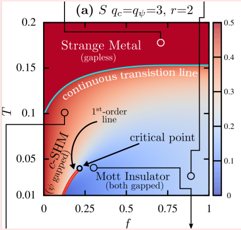
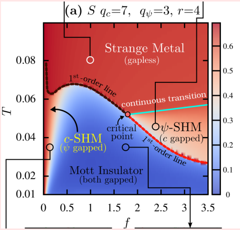
Highlights (arXiv:1703.05111)

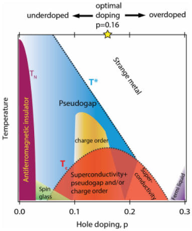
- Low temperature instabilities of coupled SYK dots
- Strange metal gives way to new phases – strange half metals, and Mott insulators in a rich phase diagram





The Question

- Several examples in condensed matter physics where non-Fermi liquid/strange metal phases undergo instabilities to produce interesting new phases...e. g., High T_c superconductors



(Vishik et al. (2015))

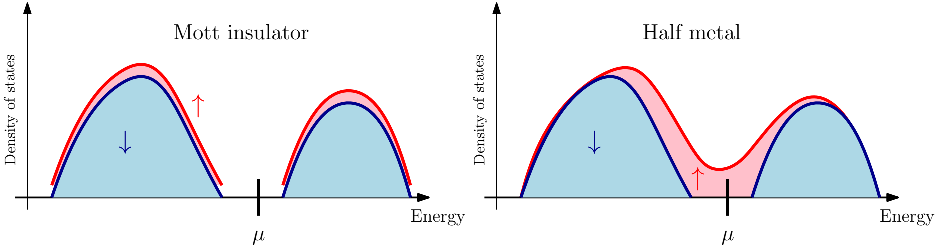
- SYK (Sachdev-Ye-Kitaev) model(Sachdev and Ye (1994), Kitaev (2015)) – solvable example of a non-Fermi liquid

Question

- Can the SYK model throw light on possible instabilities of non-Fermi liquids?
- Phase transitions in (?between) non-Fermi liquids

Recap of Essential Strong Correlation Physics

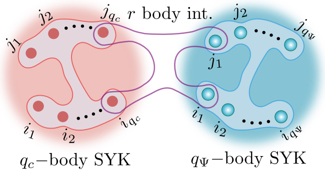
- Fermionic systems with two spin flavors (paramagnetic metal when interactions are absent)
- Strong local repulsive interactions (e. g., Hubbard model) can lead to interesting phases



- Mott insulator: Charge gapped; spins as low energy degrees of freedom (at commensurate fillings) (Lee, Wen, Nagaosa (2005))
- Half metal: One of the spin species is gapped, and the second one is gapless with a net spin polarization (Galanakis and Dederichs (2015))

Our Model: Interaction Coupled SYK Dots

- SYK dots with arbitrary q-body interactions



$$\mathscr{H} = \sum_{i_1,\ldots,i_{q_c},j_1,\ldots,j_{q_c}} H^c_{i_1,\ldots,i_{q_c};j_1,\ldots,j_{q_c}} c^\dagger_{i_{q_c}} \ldots c^\dagger_{i_1} c_{j_1} \ldots c_{j_{q_c}}$$
$$+ \sum_{\alpha_1,\ldots,\alpha_{q_\Psi},\gamma_1,\ldots,\gamma_{q_\Psi}} H^c_{\alpha_1,\ldots,\alpha_{q_\Psi};\gamma_1,\ldots,\gamma_{q_\Psi}} \Psi^\dagger_{\alpha_{q_\Psi}} \ldots \Psi^\dagger_{\alpha_1} \Psi_{\gamma_1} \ldots \Psi_{\gamma_{q_\Psi}}$$
$$+ \sum_{i_1,\ldots,i_r,\alpha_1,\ldots,\alpha_r} H^{c\Psi}_{i_1,\ldots,i_r;\alpha_1,\ldots,\alpha_r} c^\dagger_{i_r} \ldots c^\dagger_{i_1} \Psi^\dagger_{\alpha_1} \Psi_{\alpha_1} \ldots \Psi_{\alpha_r} + \text{h. c.}.$$

- Dot c with c-fermions, N_c sites, q_c-body interactions described by energy scale J_c.
- Dot Ψ with Ψ-fermions, N_Ψ sites, q_Ψ-body interactions described by energy scale J_Ψ (take $q_\Psi \leq q_c$)
- Inter-dot r-body interaction described by energy scale V
- Fraction of sites $f = \frac{N_\Psi}{N_c} \Longleftarrow$ *key parameter*
- Case $q_c = 2$, $q_\Psi = 1$ and $r = 1$ already considered (Banerjee and Altman (2017))

Analytics

- Large-N action at inverse temperature β,

$$\mathcal{S} = N\Xi = \frac{N}{1+f}\Bigg[-\frac{1}{\beta}\ln\det[-G_c^{-1}] - \frac{f}{\beta}\ln\det[-G_\Psi^{-1}] - \sum_{s=c,\Psi}(-1)^{q_s} f^{\frac{1-s}{2}}\frac{J_s^2}{2q_s}\int_0^\beta d\tau G_s^{q_s}(-\tau)G_s^{q_s}(\tau)$$
$$-\sum_{s=c,\Psi} f^{\frac{1-s}{2}}\int_0^\beta d\tau\, \Sigma_s(\tau)G_s(-\tau) - (-1)^r\sqrt{f}\frac{V^2}{r}\int_0^\beta d\tau G_c^r(-\tau)G_\Psi^r(\tau)\Bigg]$$

τ–imaginary time, G_s – Green function, Σ_s – self energy

- Self consistency condition

$$\begin{aligned}\Sigma_s(\tau) =& (-1)^{q_s+1}J_s^2 G_s^{q_s-1}(-\tau)G_s^{q_s}(\tau)\\ &+ (-1)^{r+1}(\sqrt{f})^s V^2 G_s^{r-1}(-\tau)G_{\bar{s}}^r(\tau), \quad s = c, \Psi\end{aligned}$$

- Use a "conformal ansatz" (Sachdev *PRX* (2015) and references therein)

$$G_s(\tau) = -C_s\frac{\operatorname{sgn}\tau}{|\tau|^{2\Delta_s}}, \quad s = c, \Psi$$

τ – imaginary time, Δ_s – fermion dimension, C_s – constant

Analytics...contd.

- Uncoupled dots $V = 0$

$$\Delta_s = \frac{1}{2q_s} \equiv \Delta_s^0, \quad C_s^{2q_s} = \frac{1}{J_s^2} K(\Delta_s), \quad s = c, \Psi$$

$K(x) = \frac{1}{\pi}(\frac{1}{2} - x)\tan(\pi x)$

- Analysis for $V \ll J_c, J_\Psi$: crucial parameter

$$r_\star = \frac{2q_c q_\Psi}{q_c + q_\Psi}$$

- If $r > r_\star$, coupling V is *irrelevant* on both dots

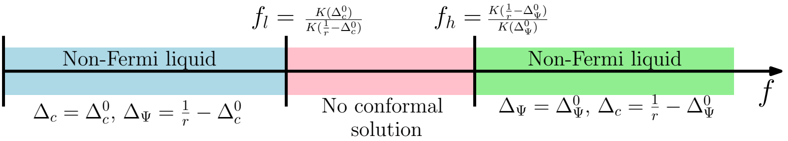
$$\Delta_s = \Delta_s^0, \quad C_s^{2q_s} = \frac{1}{J_s^2} K(\Delta_s), \quad s = c, \Psi$$

- If $r = r_\star$, coupling V is *marginal* on both dots

$$\Delta_s = \Delta_s^0, \quad C_s^{2q_s} \text{ depends on } J_s, V \text{ and } f$$

Analytics...contd.

- For $q_\Psi < r < r_\star$, V is *relevant* on one flavor and *marginal* on the other depending on the value of f



  - No conformal solution for $f_l \leq f \leq f_h$
  - Can change the nature of the non-Fermi liquid by tuning f, but a "non conformal phase" intervenes!
  - Key question: What is the nature of the intervening non-conformal phase?

- For $r < q_\Psi$, V is *relevant* on both dots

$$\frac{K(\Delta_c)}{\frac{1}{r} - \Delta_c} = f, \quad \Delta_\Psi = \frac{1}{r} - \Delta_c$$

fermion dimension is f dependent! (For further details see 1703.05111)

Numerics

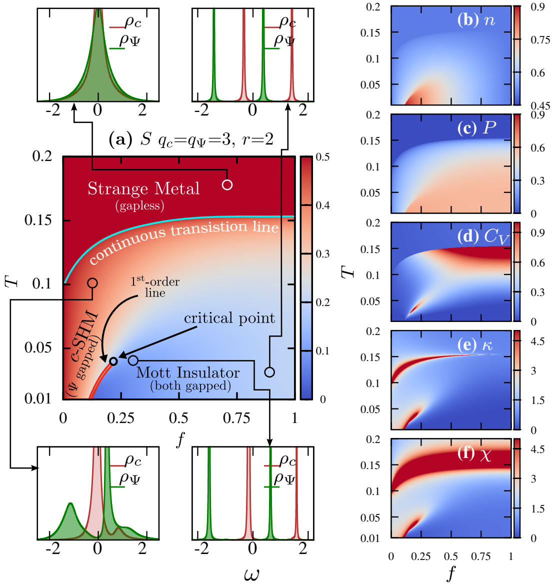
- Numerical solution of the self consistency equations

$$\begin{aligned}\Sigma_s(\tau) =&(-1)^{q_s+1} J_s^2 G_s^{q_s-1}(-\tau) G_s^{q_s}(\tau) \\ &+ (-1)^{r+1} (\sqrt{f})^s V^2 G_s^{r-1}(-\tau) G_{\bar{s}}^{r}(\tau), \quad s = c, \Psi\end{aligned}$$

- Work at fixed chemical potential $\mu = 0$
- Quantities of interest
  - Spectral function $\rho_s(\omega)$
  - Entropy S
  - Number density n
  - Polarization $P = n_c - n_\Psi$
  - Specific heat C_V
  - Compressibility κ
  - "Magnetic susceptibility" χ

Numerical Results

- $q_c = q_\Psi = 3, r = 2, J_c = J_\Psi = V = 1$



- High temperature phase is strange metal with a large entropy akin to usual SYK; here $n = 1, P = 0$ independent of f
- At a critical temperature $T_c(f)$, a second order (Landau like) transition occurs – instability of the strange metal
- Phase below T_c depends on f
- For $f \ll 1$, a c-strange *half* metal (c – SHM) emerges – c is gapless, Ψ is gapped
- For $f \approx 1$, a Mott insulator (MI) phase where both c and Ψ are gapped occurs
- A very low temperatures, a first order transition separates the c – SHM and MI; the first order line end in a critical point
- Other quantities calculated are all consistent with this picture

Physics of Instability

- Insights by considering $J_c = J_\Psi = 0, V \neq 0$, whose free energy

$$\Xi = \frac{\sqrt{f}}{1+f}\frac{V^2}{r}\int_0^\beta \mathrm{d}\tau\, G_c^r(\beta-\tau)G_\Psi^r(\tau)$$

- For $f = 1$, a particle hole symmetric solution implies

$$G_s(\beta-\tau) = G_s(\tau) \Longrightarrow \rho_s(-\omega) = \rho_s(\omega) \quad s = c, \Psi$$

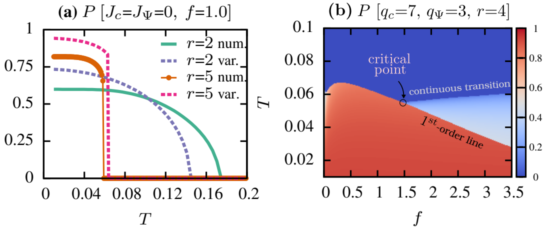
- Interaction problem viewed as two "classical strings" with long ranged interactions
- Particle hole symmetric solution – good for entropy
- Interaction energy is reduced by breaking particle hole symmetry

A Variational Ansatz

- Classical string analogy suggests a variational ansatz

$$G_s^{\text{var}}(\mathbb{i}\omega_n) = \frac{1}{\mathbb{i}\omega_n + d_s\xi}; \qquad d_{s=c} = -1, d_{s=\Psi} = 1$$

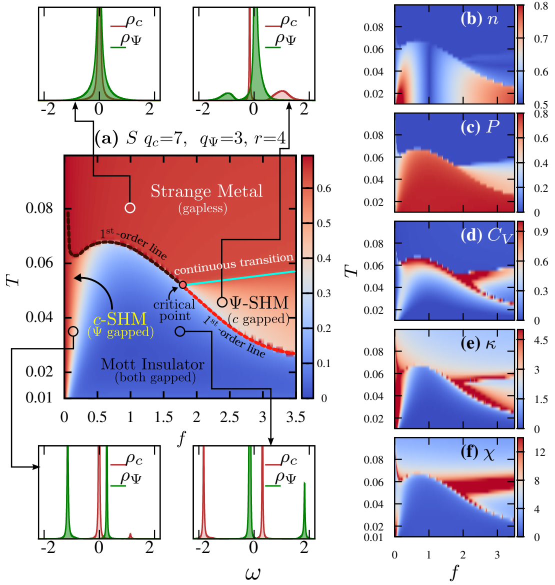
- Key results of variational study
  - For $r = 1$ there is no instability; dots must be coupled by *interactions* for the low temperature instability
  - For $J_c = J_\Psi = 0, f = 1$, instability via *second order phase transition* for $r = 2, 3$, but *first order for* $r \geq 4$! (confirmed by full numerics)



  - Promises a very rich phase diagram for $q_c = 7, q_\Psi = 3, r = 4, J_c = J_\Psi = V = 1$ with a plethora of phases, critical lines and multicritical points

Numerical Results

- $q_c = 7, q_\Psi = 3, r = 4, J_c = J_\Psi = V = 1$



- Rich phase diagram predicted by variational study is indeed found!
- SHM and MI phases appear at low temperature
- There are many phase transitions, both continuous and first order, with multicritical points

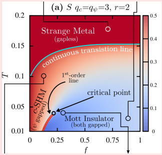
Summary

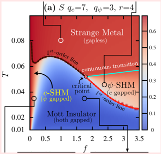
What we study

- Two SYK dots each with arbitrary q body interactions, coupled with r body interactions; key parameter, ratio f of number of sites

What we learn (1703.05111)

- Analytical result – possible to go from one type of strange metal to another by tuning f
- Numerics: The coupled strange metals are unstable at low temperatures – giving way to new phase such as strange half metals and Mott insulators





- Physical insights from a variational approach
- Future work: Realize such physics in lattice systems (see 1710.00842)