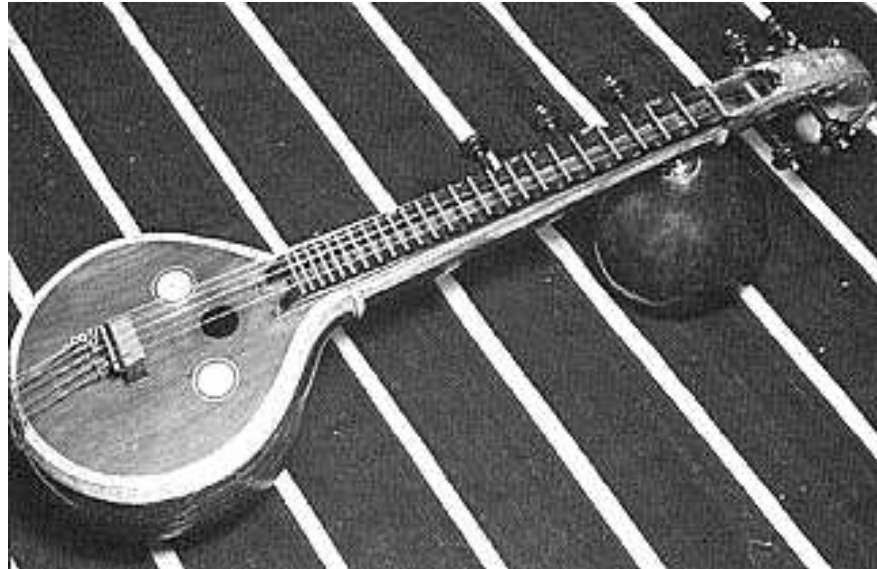




Review of Classical Mechanics

Some Questions

- Why does a tennis racket wobble when flipped along a certain axis?
- What do hear when you pluck a Veena string?



- How do you hear it, and how do you tell difference between Bhairavi and Huseni?



Plan of Review

- **Newton's Laws**
 - The Laws
 - Example: Spring Mass System
- **Hamiltonian Formulation**
 - Configuration or State of a Particle – Phase Space
 - Hamiltonian Equations
 - Example: Spring Mass System
 - Many particles
- **Continuous Systems**
 - The Veena String – Configuration
 - Newton and The Veena String

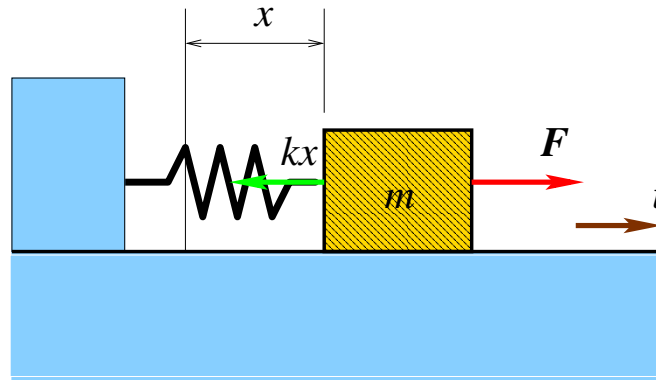


Newton's Laws

- First: Every body continues to be in a state of rest or of motion in a straight line unless compelled by an external force
- Second: $F = ma = \frac{dp}{dt}$
- Why two laws? What is force? What is mass?
- The configuration is described by a vector r
Acceleration is the second time derivative of r
- Newton's Law gives a second order differential equation for r

$$m \frac{d^2 r}{dt^2} = F$$

Newton's Law – Spring Mass System



- F – Force that you applied
- kx – Restoring force of the spring (acts in the –ve direction)
- Newton's Law

$$F - kx = m \frac{d^2x}{dt^2}$$

- A second order differential equation in x



Energy and things...

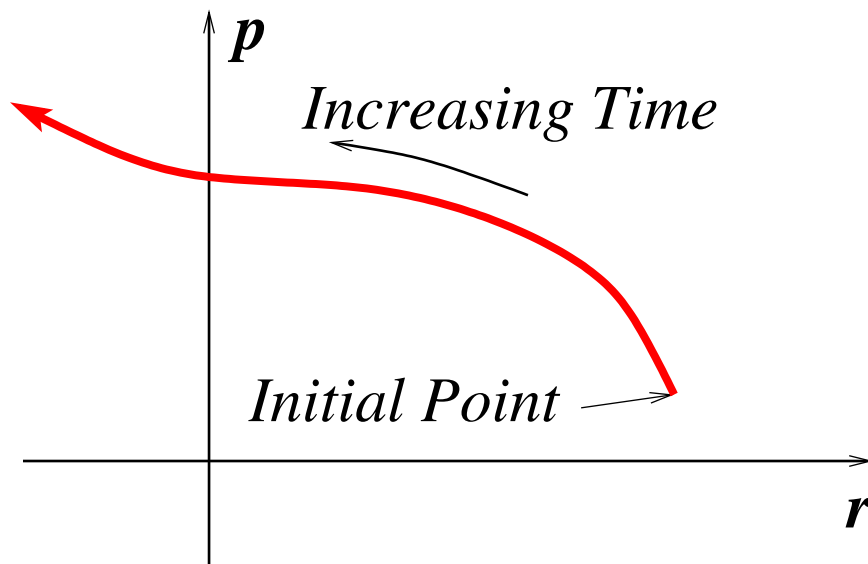
- Kinetic energy $\frac{1}{2}m\mathbf{v} \cdot \mathbf{v} = \frac{\mathbf{p} \cdot \mathbf{p}}{2m}$
- Potential Energy $V(\mathbf{r})$
- Conservative Force is derived from Potential Energy

$$\mathbf{F} = -\nabla V$$

- Total energy $\frac{\mathbf{p} \cdot \mathbf{p}}{2m} + V(\mathbf{r})$
- Total Energy is Conserved!
- Example: Roller Coaster

Hamiltonian Formulation

- Configuration (or state) of a particle is described by the pair (r, p)
- The $r - p$ space is called Phase Space
- The Hamiltonian formalism describes the evolution of the state of the particle via a trajectory in phase space





Hamiltonian Formulation

- What is the equation of the trajectory?
- The Hamiltonian function (usually equal to total energy)

$$H(\mathbf{r}, \mathbf{p}) = \frac{\mathbf{p} \cdot \mathbf{p}}{2m} + V(\mathbf{r})$$

- Hamilton's equations of motion

$$\frac{d\mathbf{r}}{dt} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{r}}$$

- Two FIRST order differential equations



Example – Free Particle

- First thing: Write down Hamiltonian

$$H(\mathbf{r}, \mathbf{p}) = \frac{\mathbf{p} \cdot \mathbf{p}}{2m}$$

- Hamilton's equations of motion

$$\frac{d\mathbf{r}}{dt} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\mathbf{p}}{m}, \quad \frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{r}} = 0$$

- Solution (initial state $(\mathbf{r}_0, \mathbf{p}_0)$)

$$\mathbf{r}(t) = \mathbf{r}_0 + \frac{\mathbf{p}_0}{m}t, \quad \mathbf{p}(t) = \mathbf{p}_0$$

- What is the phase space trajectory?



Example – 1D-Simple Harmonic Oscillator

- First thing: Write down Hamiltonian (ω – nat. freq.)

$$H(x, p) = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2$$

- Hamilton's equations of motion

$$\frac{dx}{dt} = \frac{\partial H}{\partial p} = \frac{p}{m}, \quad \frac{dp}{dt} = -\frac{\partial H}{\partial x} = -m\omega^2 x$$

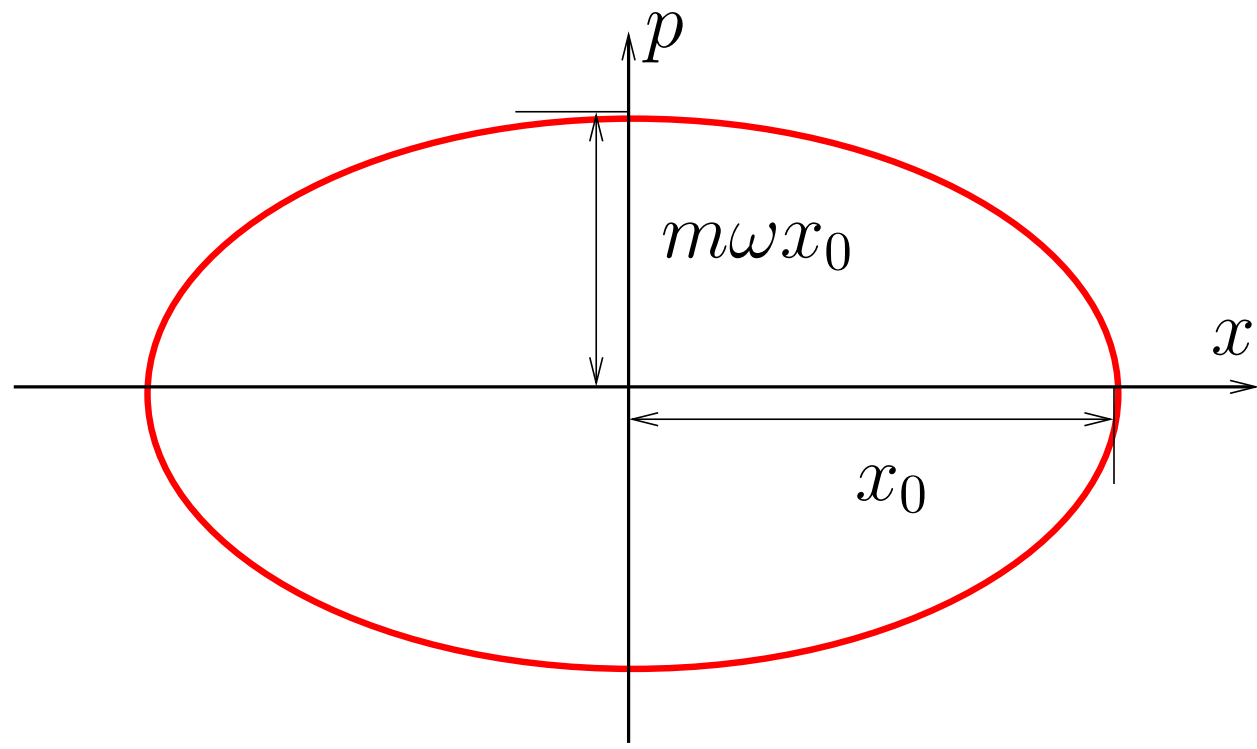
- Solution (initial state $(x_0, 0)$, “plucked”)

$$x(t) = x_0 \cos(\omega t), \quad p(t) = -m\omega x_0 \sin(\omega t)$$

- What is the phase space trajectory?



Example – 1D-Simple Harmonic Oscillator



- What is the equation of the trajectory?
- What is the arrow representing time?



Many Particle Systems

- First thing: Write down Hamiltonian (N particles)

$$H(\mathbf{r}_i, \mathbf{p}_i) = \sum_{i=1}^N \frac{\mathbf{p}_i \cdot \mathbf{p}_i}{2m} + V(\mathbf{r}_1, \dots, \mathbf{r}_N)$$

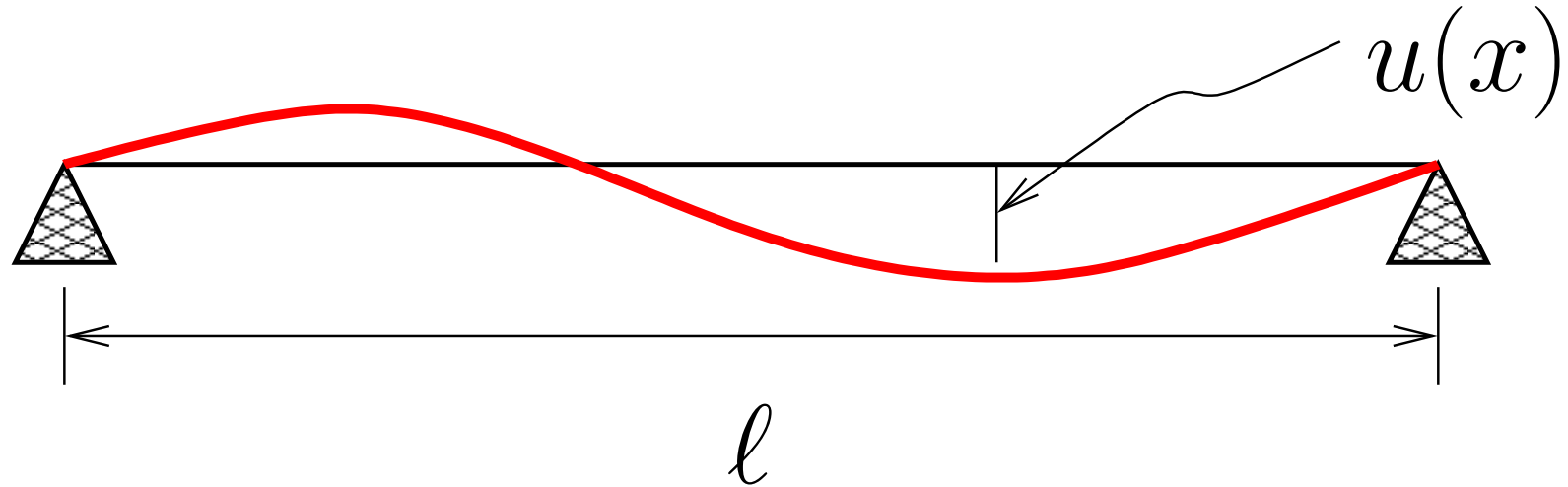
- $V(\mathbf{r}_1, \dots, \mathbf{r}_N)$ – Interaction between particles
- Hamilton's equations of motion

$$\frac{d\mathbf{r}_i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i}, \quad \frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{r}_i}$$

- What is the phase space trajectory? Well, it is a curve in $\sim 10^{23}$ dimensional phase space for atoms that make up materials!!!!

The Veena String

- How do you describe the configuration of such a string?

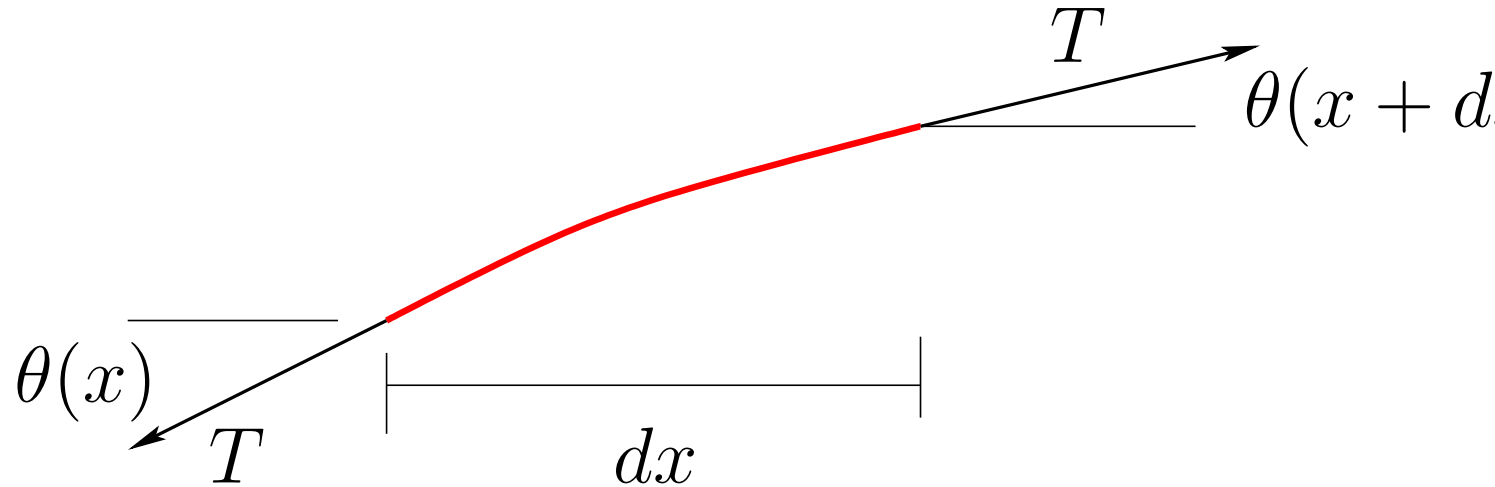


- The configuration is described by a function $u(x)$ (as opposed to a number for a particle) with $u(0) = 0$ and $u(\ell) = 0$.
- How does this configuration evolve? Need to find $u(x, t)$



Newton and The Veena String

- Veena string (ρ - "linear density", T - tension)



- Newton's Law gives

$$T\theta(x + dx) - T\theta(x) = \rho dx \frac{\partial^2 u}{\partial t^2} \quad \Rightarrow \quad T \frac{\partial \theta}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}$$
$$\theta = \frac{\partial u}{\partial x} \quad \Rightarrow \quad T \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$



What do we hear?

- Find natural frequencies

$$T \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

with $u(0, t) = u(\ell, t) = 0$

- Look for solutions of the type $u(x, t) = v(x)e^{i\omega t}$,
 ω – natural frequency(ies)

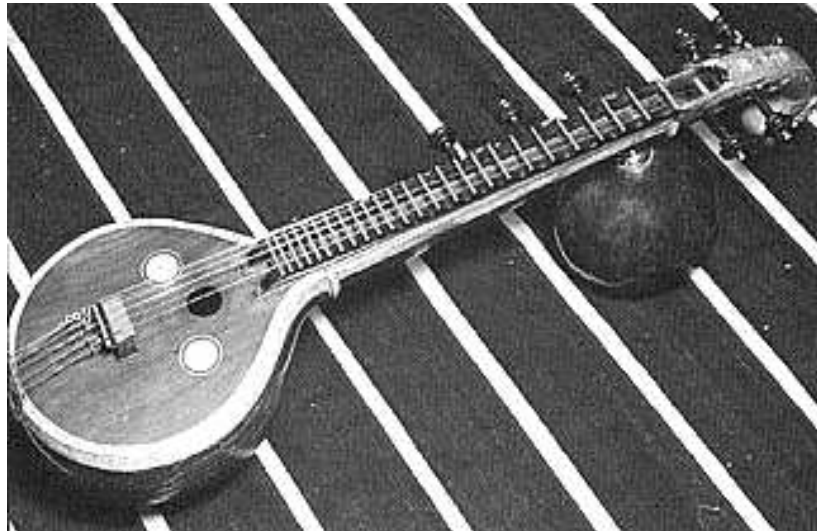
$$T \frac{d^2 v}{dx^2} + \rho \omega^2 v = 0$$

with $v(0) = v(\ell) = 0$

What do we hear? contd..

● Solution

$$v(x) = A_n \sin \sqrt{\frac{\rho}{T}} \omega_n x, \quad \omega_n = \frac{n\pi}{\ell} \sqrt{\frac{T}{\rho}}$$



● So, whats the difference between Veena and Guitar?

How do we hear?

- The Ear



- The amazing Cochlea!
- Bhairavi vs Huseni?



Summary

- **Newton's Laws**
- **Hamiltonian Formulation**
- **Phase Space**
- **Continuous Systems**