



Quantum Mechanics - The Dirac Way



Questions...

- What happens in higher dimensions?
- What is Dirac notation? What are Bra, Ket vectors etc...

Higher Dimensions

- Essentially no difference!
- State is described by $\Psi(\mathbf{r}, t)$
- Position operator R , $\langle R \rangle = \int \Psi^*(\mathbf{r}, t) \mathbf{r} \Psi(\mathbf{r}, t) d\mathbf{r}$
- Momentum operator $P = -i\hbar \nabla$
- Commutation relation $[R_i, P_j] = i\hbar \delta_{ij}$!
- Schrödinger equation $i\hbar \frac{\partial \Psi}{\partial t} = H \Psi$
- Will look at examples later



The Dirac Notation

- Why? Because, it is compact, clear and widely used!
- Main idea: Write state of particle without reference to particular basis
- A general state is represented as $|\psi\rangle$ called a “ket” vector
- Associated with every state $|\psi\rangle$ there is something called the dual $\langle\psi|$ called the “bra” vector
- The inner product of two states is given as $\langle\phi|\psi\rangle$
- The inner product satisfies $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$



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The Dirac Notation

- Action of an operator A is represented as $A|\psi\rangle$
- Suppose $|\phi\rangle = A|\psi\rangle$, then $\langle\phi| = \langle\psi|A^\dagger$
- The expected value of an operator is given by $\langle\psi|A|\psi\rangle$ when the particle is in state $|\psi\rangle$
- The eigen ket of A with eigenvalue a is denoted as $|a\rangle$, with $A|a\rangle = a|a\rangle$
- Notation may be (ab)used differently; for example $H|E_n\rangle = E_n|E_n\rangle$ is usually written as $H|n\rangle = E_n|n\rangle$



The Dirac Notation

- If $\{|n\rangle\}$ are eigen kets of a Hermitian operator (say H), they form a complete set
- Any state $|\psi\rangle$ can be written in terms of $\{|n\rangle\}$
- $|\psi\rangle = \sum_n a_n |n\rangle$ with $a_n = \langle n|\psi\rangle$...just like for “usual” vectors!
- If complete set is continuous then $|\psi\rangle = \int \langle\alpha|\psi\rangle |\alpha\rangle d\alpha$
- Similarly, operator A can be written as $A = \sum_{m,n} A_{mn} |m\rangle \langle n|$ with $A_{mn} = \langle m|A|n\rangle$
- In particular, $I = \sum_n |n\rangle \langle n|$

How's all this related to what we have done?

Stick to 1D

- What we have done is in position basis...we have written everything in terms of $|x\rangle$, i. e., the eigenvectors of the position operator X (recall $X|x\rangle = x|x\rangle$)
- The “wavefunction” $\psi(x)$ is nothing but the position representation of the state $|\psi\rangle$ such that $|\psi\rangle = \int \langle x|\psi\rangle |x\rangle dx$...thus $\psi(x) = \langle x|\psi\rangle$!
- Now $\langle x|P|\psi\rangle = -i\hbar \frac{\partial \psi}{\partial x}$
- Similarly, $\langle x|H|\psi\rangle = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$
- What is $\langle x|p\rangle$?



Summary

- **Generalization to higher dimensions...straightforward!**
- **Dirac notation, independent of choice of basis**
- **Will get used to it as we go along!**