



MR301: Concepts in Materials Science I
Materials Research Centre, IISc Bangalore
Semester I, 2005–2006

PROBLEM SET 2

2/1. **Independence day!** Which of the following sets of vectors are linearly independent? Clearly show the proofs of your claim.

- (a) $\{\mathbf{i}, \mathbf{j}\}$
- (b) $\{\mathbf{i} + \mathbf{j}, \mathbf{i} - \mathbf{j}\}$
- (c) $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$
- (d) $\{2\mathbf{i} + \mathbf{k}, 2\mathbf{j} - \mathbf{k}, 8\mathbf{i} + 8\mathbf{j}\}$
- (e) $\{1, x\}$ (vectors are elements of $C[0, 1]$)

2/2. **Eigenvalues and Eigenvectors:** Find eigenvalues and eigenvectors of $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

2/3. **Orthogonality:** For each of the vectors given below in $C[0, 1]$, find an orthogonal vector

- $\frac{1}{2} - x$, $\sin(\pi x)$, x^2 , e^x

2/4. **Function spaces:** Which of the following are vector spaces under scalar multiplication and addition operation defined in class

- (a) The set of all continuous functions defined on $[0, 1]$ which have values of 0 at both $x = 0$ and $x = 1$.
- (b) The set of all continuous functions defined on $[0, 1]$ which have values of 0 at $x = 0$ and 1 at $x = 1$.
- (c) The set of all continuous functions defined on $[0, 1]$ which have values of 0 at $x = 1$ and 1 at $x = 1$.
- (d) $C_{0,0}^\infty[0, 1]$ (Defined in the notes).

2/5. **Eigenvalues and eigenvector...“ei”gain!** Find eigenvalues and eigenvectors of operators $D^2 + 1$ and $D^2 - 1$ defined on $C_{0,0}^\infty[0, 1]$.

2/6. **“Really Complex” problem!** Consider the set of all *bounded*, continuous complex valued functions defined on the real line. Is this a vector space under complex scalar multiplication and addition of functions defined in class? If it is a vector space, find the eigenvalues and eigenvectors of the operator D , if they exist. Are the eigenvalues real? (You should find that they are not!) How do you make them real? Understanding this math is key to understanding a “free particle” in 1D quantum mechanics.