



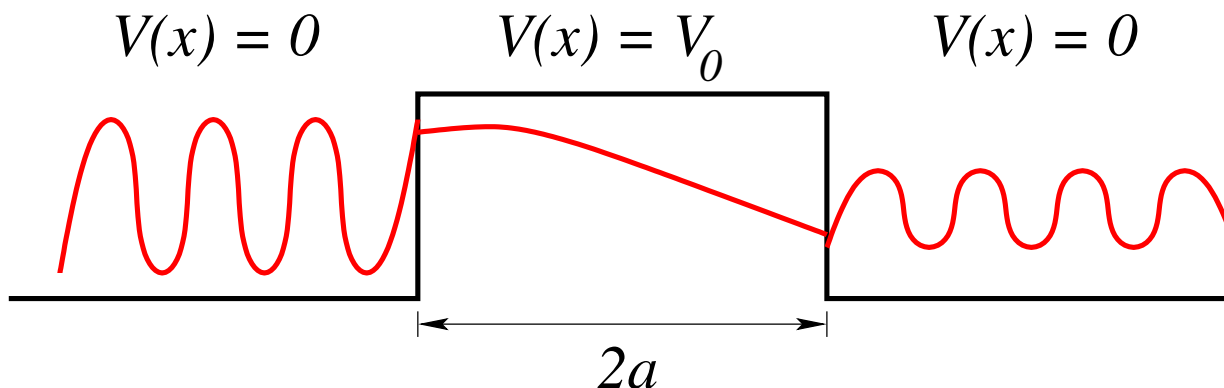
Tunnels and Wells



Of course, the questions..

- 1. How does an alpha particle escape a nucleus?**
- 2. Whats with STM? (Scanning tunneling microscope)**
- 3. And quantum wells?**
- 4. Ramsauer effect – alpha particles incident on inert gas atoms show “resonances” in transmission!**
- 5. Bound states? Metal-insulator transitions!**

The Great Barrier Tunneler!



- Model (What do you expect classically?)
- Schrödinger Recipe: Say energy state is E

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = E\psi, \quad |x| \geq a; \quad -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V_0\psi = E\psi, \quad |x| < a$$

- Is $E < V_0$ possible? What happens?
- Two length scales (a and $\sqrt{\frac{\hbar^2}{2mV_0}}$)!!



Barrier Tunneling

- Think of a state when electron is incident from left
- State to the left of the barrier $x < -a$: superposition of plane wave “traveling” to right and a “reflected wave” $\psi_l(x) = e^{ikx} + Re^{-ikx}$, $k = \sqrt{\frac{2mE}{\hbar^2}}$
- To the right of the barrier a wave traveling to right $\psi_r(x) = Te^{ikx}$
- Restrict to case when $E < V_0$
- The state in the barrier is $\psi_b(x) = Ae^{qx} + Be^{-qx}$
($q = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$) – does not correspond to a wavelike state



Barrier Tunneling

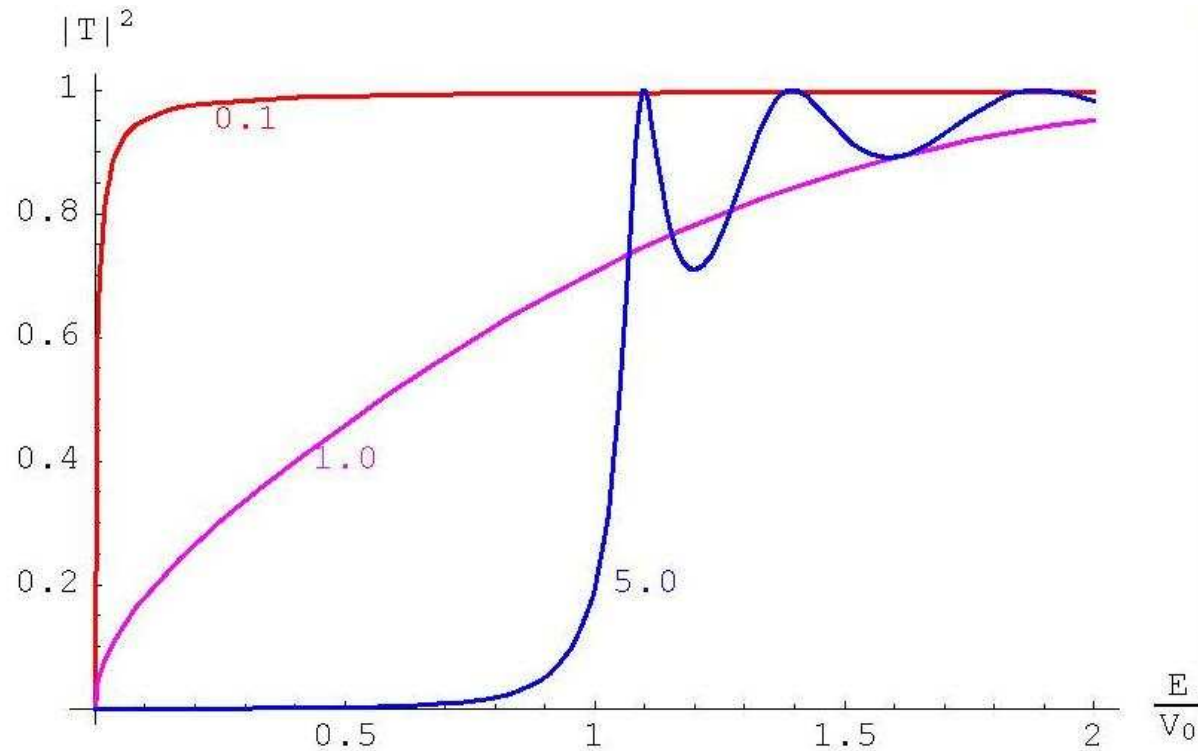
- How to determine R, T, A, B ? Of interest: T !
- Use conditions

$$\begin{aligned}\psi_l(-a) &= \psi_b(-a), & \psi'_l(-a) &= \psi'_b(-a) \\ \psi_b(a) &= \psi_r(a), & \psi'_b(a) &= \psi'_r(a)\end{aligned}$$

four equations for four unknowns! All OK!

- Solution $T = \frac{4ie^{2a(-ik+q)}kq}{(1-e^{4aq})q^2+2i(1+e^{4aq})kq-(1-e^{4aq})k^2}$
- The quantity $|T|^2$ is probability of transmission...upshot $|T|^2 \sim e^{-4aq}$!
- For a given V_0 , how does $|T|^2$ depend on E , for various values of $a/\sqrt{\frac{\hbar^2}{2mV_0}}$?

Barrier Tunneling



Curves for various $a/\sqrt{\frac{\hbar^2}{2mV_0}}$ indicated

- For small a the electron does not feel the barrier, for large a there is very little transmission
- Note “resonances” for large values of a , when $E > V_0$

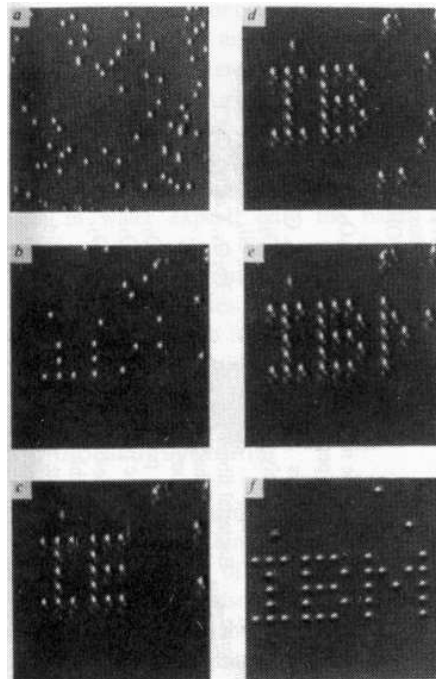


Tunneling in Real Life

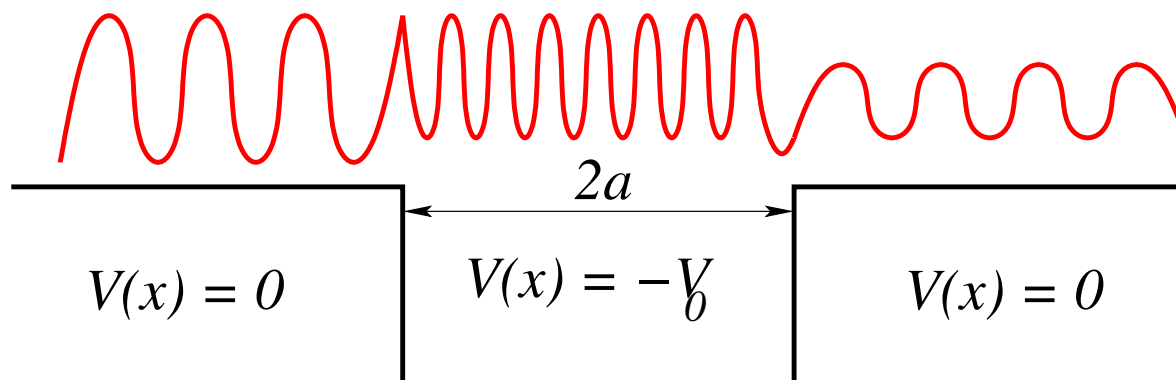
- **Alpha particles:** Held to nucleus by very strong nuclear forces. Barrier height can be of many eVs!! Very high kinetic energy (confinement in 10^{-15} m)!! $|T|^2$ is quite small (in fact, can be shown to be $\sim e^{-90}$ (!!!!!)), the number of “attempts” to jump out are large (of the order of 10^{21} per second...probability of decay is 10^{-11} per year!!)
- **Ohmic contacts between metals and semiconductors:** Need to have linear $I - V$ characteristics for contacts...problem...contact barrier! Solution: Try to make a as small as possible by heavily doping semiconductor near contact...depletion zone size comes down and electrons tunnel across!

Tunneling in Real Life

- Field emission devices...pull out electrons using voltage—flat panel displays
- STM...maintain a constant tunneling current (at constant voltage)
by adjusting distance...maps out topography of surface!



Well, What Now?...A Well.. Well! Well!



- Model (What do you expect classically?)
- Schrödinger Recipe: Say energy state is E

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = E\psi, \quad |x| \geq a; \quad -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - V_0\psi = E\psi, \quad |x| < a$$

- What happens when $E > 0$ Is $E < 0$ possible?

Quantum Well

- Think of a state when electron is incident from left
- State to the left of the well $x < -a$: superposition of plane wave “traveling” to right and a “reflected wave”

$$\psi_l(x) = e^{ikx} + Re^{-ikx}, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

- To the right of the well a wave traveling to right

$$\psi_r(x) = Te^{ikx}$$

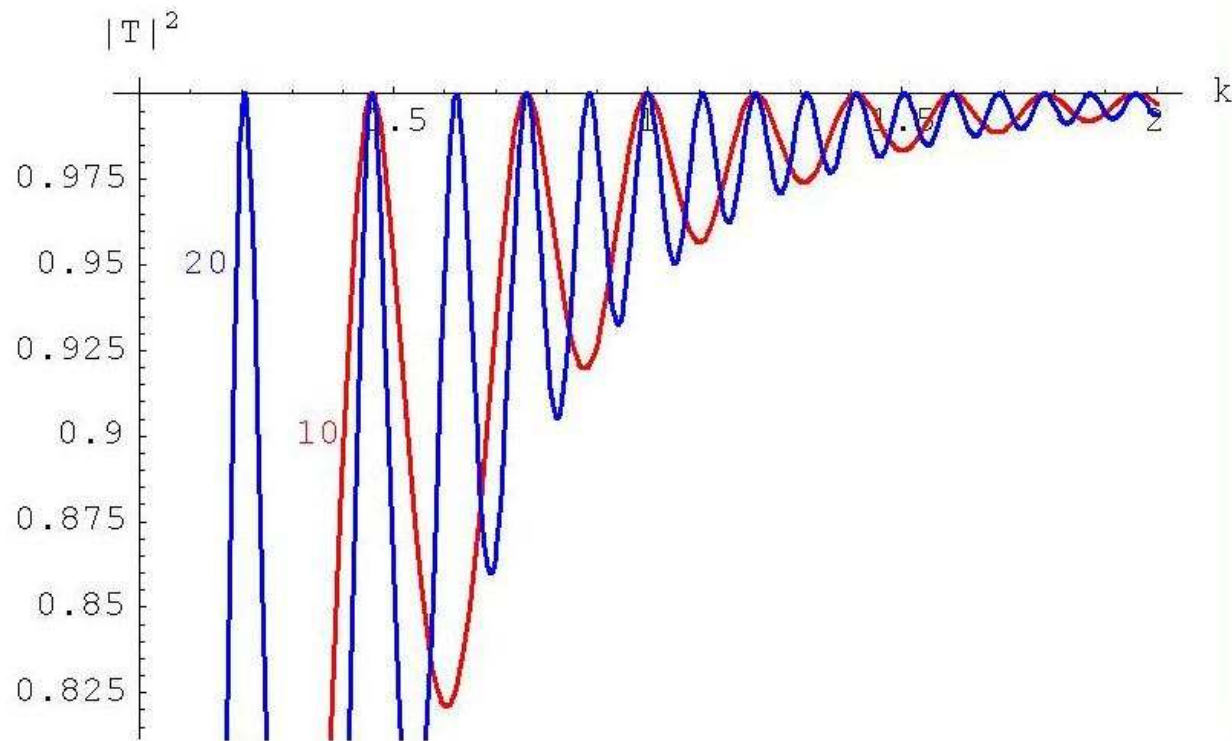
- Restrict to case when $E > 0$

- The state in the well is $\psi_w(x) = Ae^{iqx} + Be^{-iqx}$

$(q = \sqrt{\frac{2m(E+V_0)}{\hbar^2}})$ – does correspond to a wavelike state of shorter wavelength!

- Solve for T in exactly the same way as the barrier problem!

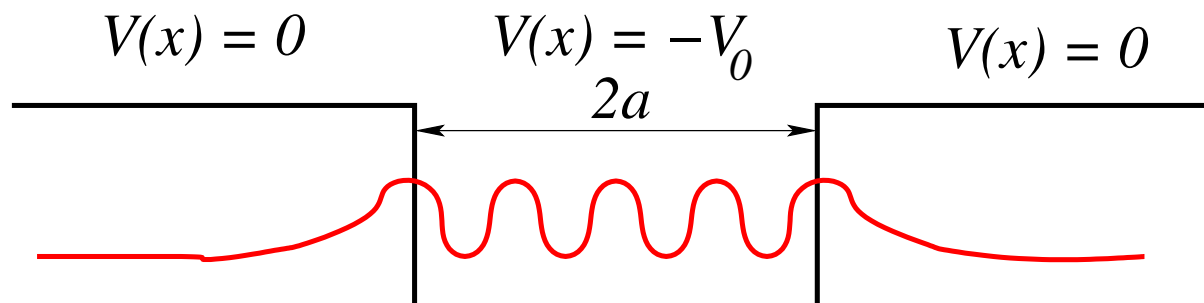
Quantum Well



Curves for various $a/\sqrt{\frac{\hbar^2}{2mV_0}}$ indicated

- There are particular values of k at which $T = 1...$ "Resonances" ...Why?
- Roughly explains the Ramsauer effect!

Quantum Well – Bound States



- States with $-V_0 < E < 0$ are possible! “Bound states”!

- Wavefunction $(q = \sqrt{\frac{2m|E|}{\hbar^2}}, k = \sqrt{\frac{2m(E - V_0)}{\hbar^2}})$

$$\psi_l(x) = Ae^{qx}, \quad \psi_w(x) = Be^{ikx} + Ce^{-ikx}, \quad \psi_r(x) = De^{-qx}$$

- Main idea: Wavefunction decay exponentially outside the box!
- How to determine allowed E s?



Quantum Well – Bound States

- Use exactly same ideas as before

$$\psi_l(-a) = \psi_w(-a), \quad \psi'_l(-a) = \psi'_w(-a)$$

$$\psi_w(a) = \psi_r(a), \quad \psi'_w(a) = \psi'_r(a)$$

- Main point: Homogeneous equations for A, B, C, D
- Condition of non-triviality gives possible energy states

$$\sqrt{\frac{|E|}{V_0 - |E|}} = \tan \left(\sqrt{\frac{2m(V_0 - |E|)a^2}{\hbar^2}} \right) \quad n \text{ even}$$

$$\sqrt{\frac{|E|}{V_0 - |E|}} = -\cot \left(\sqrt{\frac{2m(V_0 - |E|)a^2}{\hbar^2}} \right) \quad n \text{ odd}$$

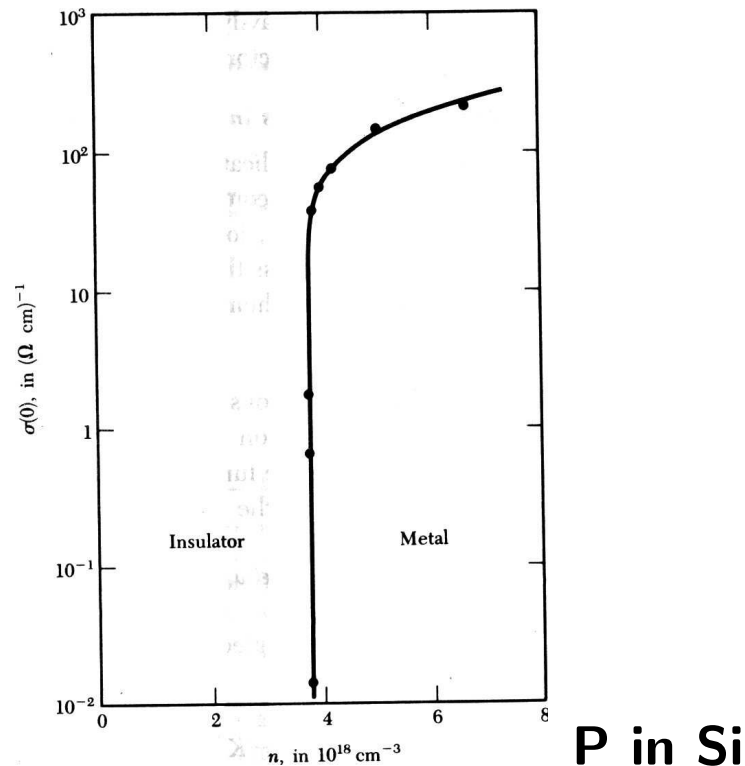


Bound States...Big Deal?

- Whats the big deal?
- The “simplest” model of an atom!!
- Bound state are always possible in 1D (and 2D..I am not sure about this) for “attractive” potentials
- Quantum wells are used for many purposes... Infrared sensor, lasers etc. etc..

Bound States...Big Deal?

- In 3D there are attractive potentials for which there are no bound states! For example, $V(r) \sim \frac{e^{-\xi r}}{r}$, there is no bound state for $\xi > \xi_c$...crucial to understand metal-insulator transitions (Mott)





Summary

- **Tunneling** : Transmission probability goes as $e^{-2\ell q}$ where ℓ is the size scale of the barrier
- **Quantum wells**: “Resonances”
- **Bound states**: Again useful for many things!